# الجمهورية الديمقراطية الشعبية الجزائرية République Algérienne Démocratique et Populaire وزارة التعليم العالي والبحث العلمي

Ministère de l'Enseignement Supérieur et de la Recherche Scientifique

المدرسة العليا في العلوم التطبيقية بالجزائر Ecole Supérieure des Sciences Appliquées Alger المدرسة الوطنية المتعددة التقنيات Ecole Nationale Polytechnique





Département du second cycle

# Mémoire de Fin d'Etudes

En vue de l'obtention du diplôme de MASTER

Filière: Electrotechnique

Spécialité: Traction électrique

Thème:

# State of the art on the Model Predictive Control

Présenté par : Mekhilef Aymen

Abdelmounaim

Encadré par : Dr. Benachour Ali Co-encadré par : Dr. Dali Ali

Soutenu le : 02/09/2020, Devant le jury composé de :

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M Deboucha Abdelhakim Examinateur

Monôme  $N^{\circ}: 26$  / Master / 2020

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#### ملخص:

الهدف الرئيسي من هذه الرسالة هو إجراء استطلاع حول التحكم التنبئي باستخدام نموذج (MPC). تبدأ الدراسة بتاريخ موجز لتكنولوجيا MPC الصناعية وتطورها من حيث الجوانب النظرية. تم تقديم وشرح خوار زمية تحكم MPC عامة. بعد ذلك ، يتم التحقيق في تطبيق MPC في إلكترونيات الطاقة. لإنهاء هذا الاستطلاع ، يتم شرح وتقديم بعض التطبيقات المعينة لـ MPC كلمات مفتاحية:

التحكم التنبؤي باستخدام نموذج (MPC)، دالة التكلفة، تحكم غير خطي.

#### Abstract:

The main objective of this thesis to conduct a survey on Model Predictive Control (MPC). The survey begins with a brief history of industrial MPC technology and its development in terms of theoretical aspects. A general MPC control algorithm is presented and explained. Then, the application of MPC in power electronics is investigated. To finish this survey, some particular applications of MPC are explained and presented.

#### Key words:

Model Predictive Control (MPC), cost function, non-linear control.

#### Résumé:

L'objectif principal de cette thèse est de mener une étude sur la commande à base de modèle (MPC). L'étude commence par un bref historique de la technologie de MPC dans l'industrie et de son développement en termes d'aspects théoriques. Un algorithme de contrôle MPC général est présenté et expliqué. Ensuite, l'application du MPC dans l'électronique de puissance est étudiée. Pour terminer cette thèse, certaines applications particulières de MPC sont expliquées et présentées.

#### Mots clès:

Commande predictive à base du modèle (MPC), function de coût, commande non-linéaire.

#### Acknowledgment

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My heartfelt thanks to all the teachers and to all the staff of the Higher School of Applied Sciences of Algiers especially those from the Electrical Engineering department, Dr. Amar Hamache and Dr. Abdelhakim Debboucha in particular.

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I would like to thank everyone who helped me with my studies from the very first start.

# Dedication

In the memory of my grandfather, Seridi Ali, the foremost seeking knowledge supporter whom I have ever known,

I dedicate this humble work to my loving parents, who through thick and thin, have been and still are there for me,

To my sister,

To my family,

To my advisor,

To my co-advisor,

To my project partner,

To my friends.

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#### LIST OF SYMBOLS AND ACRONYMS

space vector modulation
 PWM : Pulse-width modulation
 PI : Proportional-Integral

MAC : Model algorithmic controlIDCOM : identification and command

FIR : finite impulse response
TSR : truncated step response

**GPC** : Generalized Predictive Control

MC : matrix converter

DMC : direct matrix converter
IMC : indirect matrix converter

**2LVSI** : two-level voltage-source inverter

MPC : model predictive control

MPCC : model predictive current controlMPTC : model predictive torque controlCCS-MPC : continuous control set MPC

FCS-MPC: finite control set MPC

M2PC : modulated MPC

NMPC : nonlinear model predictive control

C/GMRES: continuation and generalized minimum residual method

**THD** : total harmonic distortion

**RL** : resistive-inductive

#### I. Introduction

Several control schemes have been proposed for the control of power converters and drives. Among these, hysteresis and linear controls with pulse-width modulation (PWM) are the most established in the literature [1]–[3]. However, with the development of faster and more powerful microprocessors, the implementation of new and more complex control schemes became possible [4]–[6]. Some of these new control schemes for power converters include fuzzy logic, sliding mode control, and predictive control [3].

Model predictive control (MPC) has been widely adopted in industry as an effective means to deal with multivariable constrained control problems [7]. Although the ideas of MPC were developed in the 1960s as an application of optimal control theory, industrial interest in these ideas started in the late 1970s [8]. Since then, MPC has been successfully applied in the chemical process industry, where time constants are long enough to perform all the required calculations [9].

This survey begins with a historical background and the development of MPC, where the first applications of this control strategy are described alongside the major developments of its theoretical aspects

MPC describes a wide family of controllers, not a specific control strategy. Various applications of MPC of power converters and other predictive control schemes are studied in this survey.

#### II. Historical background

The first applications of MPC in the industry segment can be traced back to the processing industry (petrochemical industry in particular) [7], [8]. The idea was published in the late 1970s in two papers: "Model Predictive Heuristic Control" where the authors described successful applications of the control strategy and reported successful applications to a dozen large-scale industrial processes including a fluid catalytic cracking column and "Dynamic Matrix Control" where engineers from Shell reported applications to a fluid catalytic cracker [10], [11].

However, earlier applications can be seen in some computer-based supervisory control in petrochemical industries, which date back to 1950s [7], [12] as shown in Figure 1. As an example, the data from a fluid catalytic cracking unit at El Segundo were sent via teletype to an IBM 7090 mainframe computer, located in San Francisco. The optimal process settings were computed and sent back to El Segundo every 15-20 minutes, which were then implemented manually by the operators [7], [12]. After this approach showed successful results, on-site process computer was installed to remove the need for the telecommunication and automate the adjustments [7].

But the high cost of such computer-based control at the time, kept it from spreading within the process industries [9]. In the meantime, the idea of the MPC was still being reported in the literature [10], [11].

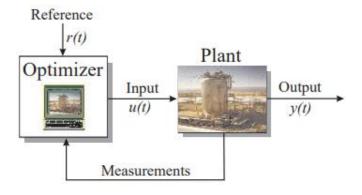


Figure 1: Computer-based supervisory control

With the advancement in microprocessor technology in the mid-70s and the research papers published around that time, the use of model-based computer control spread rapidly across refineries and petrochemical plants in the Western world [7], [12].

Thanks to the diverse companies that used this type of control, different approaches immerged like Model algorithmic control (MAC) and identification and command (IDCOM) [13]. And eventually, the community agreed to the denomination that is Model Predictive Control (MPC). These algorithms were heuristic in nature. They employed time-domain response-based models, e.g. finite impulse response (FIR) and truncated step response (TSR) and were completely deterministic without any explicit disturbance model, and lacked stability guarantees and systematic tuning guidelines [7].

As mentioned before, despite the relatively few applications of the computer-based supervisory control due to the high cost of computations back then, the adaptive control independently from the process industry community saw a rise of its own version of MPC called Generalized Predictive Control (GPC) [7]. The GPC was intended to offer a new alternative to the self-tuning regulator [14], [15] in contrast to the previous Dynamic matrix control that was conceived to handle multivariable constrained control problems typical of the oil and petrochemical industries [10]. The GPC employed a transfer function model, which makes the application of multivariable control problems quite hard and the inclusion of constraints was not possible [7]. As a result, GPC went largely unnoticed by the industry.

During the late 1980's, researchers founded a theoretical approach for the MPC: the state-space representation model in continuous time. However, discrete-time models are required for implementation.

Sys: 
$$\begin{cases} x[k+1] = Ax[k] + Bu[k] \\ y[k+1] = Cx[k] + Du[k] \end{cases}$$
 (I.1)

Sys is the model of the discrete model of the system described by the linear discrete-time difference equations where:

- $x[i] \in \mathbb{R}^n$  is the an *n*-column vector that contains the state variables and *n* is the number of states;
- $u[i] \in \mathbb{R}^m$  is the an m-column vector that contains the control inputs and m is the number input variables;
- $y[i] \in \mathbb{R}^p$  is the a p-column vector that contains the output variables and p is the number output variables.

The cost function, or the optimization criterion, is given by:

$$J = x^{T}[N_{p}]Px[N_{p}] + \sum_{i=1}^{N_{p}-1} [x^{T}[i]Qx[i] + u^{T}[i]Ru[i]]$$
 (I.2)

J is the cost function and P, Q and R are its weighting matrices of appropriate dimensions. The cost function depends on the inputs, the states, the length of the control horizon and the length of the prediction horizon, the notations x and u are used to distinguish the predicted state and computed input from the actual state x and implemented input u.

At each sample time, the control input is determined by solving the following optimization with the initial state x[0] set equal to the measured (or estimated) state value:

$$\min_{\mathbf{U}\triangleq\{u[k]\}_{k=0}^{k=N_m-1}}J(\mathbf{U},x[k],N_p,N_m) \tag{I.3}$$

Where  $N_p$  denotes length of the prediction horizon and  $N_m$  denotes the length of the control horizon.

According to [7] and [16], this representation assures the following:

- Stability: The stability can be proven using the fact that the optimal cost function  $J_{\infty}(x[k])$  (infinite horizon) qualifies as a Lyapunov function. Though in practice, the length of the prediction and control horizon must be finite to ensure feasibility. Alternate stability conditions and stabilization techniques have been reported in the literature [17]–[20].
- Robustness: Most control systems are subject to uncertainties, model mismatch and/or unknown disturbances. However, MPC, being a feedback control method, has some inherent robustness since no unmeasured disturbance is acting on the system [16];
- Nonlinear control: Most of the stability results for the constrained linear systems apply to nonlinear systems without modification [21], [22].

To summarize, more standardized and mathematical properties like stability were successfully established [7], [12]. Meanwhile, in industries, applications continued to flourish and small startup vendors got bought out by household names [7]. An important attraction of MPC is the flexibility that the use of on-line optimization affords in specifying the performance measure and constraints.

After that and once again, thanks to the technological advancement, a major shift was seen in the applications of the MPC and interest in the application of MPC in power electronics has increased considerably over the last decade. A significant number of applications involving motor drives, renewable energy applications, energy storage applications [3], [5], [6], [23]–[27] and vehicle traction control [28] are now being reported in the literature.

#### III. Model Predictive Control

In general, the main characteristic of predictive control is the use of an explicit model of the system for predicting the future behavior of the controlled variables based on measurements that reflect on the current state of the system [3], [8]. This information is used by the controller to obtain the optimal actuation, according to a predefined optimization criterion.

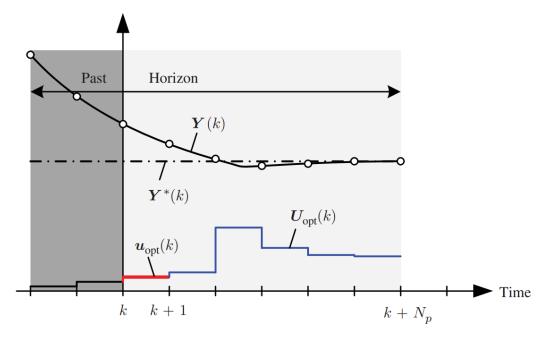


Figure 2: Prediction horizon

Where  $N_p = 6$  denotes the prediction horizon. The optimal sequence of controlled variables  $\boldsymbol{U}_{\mathrm{opt}}$  is chosen such that the predicted output sequence  $\boldsymbol{Y}$  tracks the output reference  $\boldsymbol{Y}^*$ . Out of the sequence  $\boldsymbol{U}_{\mathrm{opt}}$ , only the first element  $\boldsymbol{u}_{\mathrm{opt}}$  is applied to the system during one sampling period.

The key element of the MPC is the use of a model of the system for predicting the future behavior of the controlled variables in a predefined horizon in time. This predictive future behavior is used by the controller to obtain the optimal actuation configuration, according to a predefined optimization criterion, which is the "cost function". Then, the optimal actuation is obtained by minimizing this cost function.

The working principle of MPC can be described by the following steps:

1. Measure and/or estimate the controlled variables,  $c_k$ ;

- 2. Predicted the behavior of the controlled variables for each valid switching state of the converter,  $c_{k+1}$ ;
- 3. Evaluate the cost function and choosing an optimal actuation,  $x_{opt}$ ;
- 4. Select, store and apply of the optimal state that minimizes the cost function at the next sampling instant  $T_s$ .

Using MPC it is possible to avoid the cascaded structure which is typically used in a linear control scheme, obtaining very fast transient responses.

Nonlinearities in the system can be included in the model, avoiding the need to linearize the model for a given operating point, and improving the operation of the system for all conditions. It is also possible to include restrictions on some variables when designing the controller to best satisfy the scope statement.

In a control system it is important to reach a compromise between reference following and control effort to ensure the feasibility of the algorithm implementation. In power converters and drives, the control effort is related to the voltage or current variations, the switching frequency, or the switching losses [3]. Using predictive control, it is possible to consider any measure of control effort (switching frequency) in the cost function, in order to reduce it using what is called a "soft constraint" [17].

Other type of constraint often has to be considered, like current limitations in motor drives [29] to ensure the stability of the system, usually formulated as follows [3]. Essentially penalizing the switching state that violates a certain predefined condition.

$$J = \begin{cases} \infty, & \text{if } |c_p| > c_{max} \\ J, & \text{if } |c_p| \le c_{max} \end{cases}$$
 (I.4)

Where  $c_p$  denotes the predicted value of the controlled variable,  $c_{max}$  is the maximum allowed value for the controlled variable and J is the cost function.

#### III.1. Model Predictive Control schemes used in power electronics

Predictive control is a very wide class of controllers that have found rather recent application in the control of power converter. On the basis of modulation and switching frequency, the predictive control can be subdivided into four parts [3], [30] as shown in Figure 3.

The deadbeat control uses the model of the system to calculate, once every sampling period, the required reference voltage in order to reach the reference value in the next sampling instant, hence the need of a modulation which requires a fixed switching frequency, and it lacks the possibility of constraints inclusion [30].

Hysteresis-based predictive control strategies try to keep the controlled system variables between the boundaries of a hysteresis area or space, while in trajectory-based control, the principle is to force the system's variables onto precalculated trajectories [3], they both don't require a fixed switching frequency since there is no modulation needed because the required voltage is calculated based on suitable error boundaries.

MPC, also referred to as receding horizon control, is the only control technique which has been extremely successful in practical applications in recent decades [30], recent applications have been mentioned in historical background. An attractive feature of MPC is that it can handle general constrained nonlinear systems with multiple inputs and outputs in a unified and clear manner [3], [30]. MPC can be further subdivided into two categories, continuous control set MPC (CCS-MPC) which requires a fixed switching frequency and finite control set MPC (FCS-MPC) which doesn't.

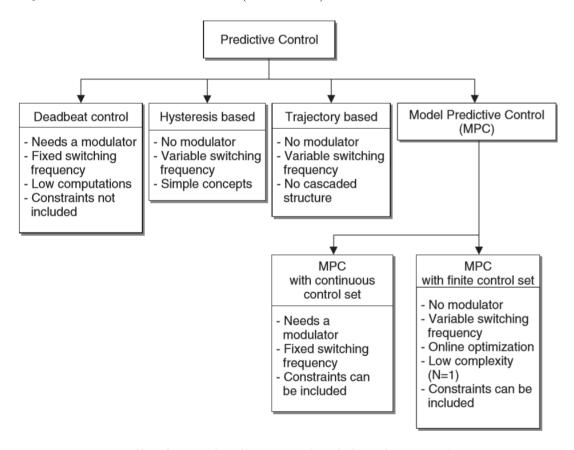


Figure 3: Classification of predictive control methods used in power electronics

#### III.1.1. Model Predictive Control of Matrix Converters

The matrix converter (MC) is a power converter topology that is capable of feeding an AC-load by an AC-source directly without the need of a storage capacitor. Hence, this configuration is appropriate for applications in which the converter volume and weight must be minimized. The MC topology presents many advantages over the conventional cascaded rectifier—inverter structure such as controllable power factor at the source side, sinusoidal waveforms at the load and input side with low harmonic content and natural, bi-directional power transfer.

The most relevant features of a MC are [3]:

- 1. The power circuit is compact.
- 2. It delivers voltages and currents to the load with high quality and without restriction on the frequency.
- 3. It can generate sinusoidal input current and operate with unity power factor.
- 4. It allows power to flow from the source to the load and in the opposite direction. This means it is very suitable for regenerative loads.

A considerable number of applications of MPC have been proposed for DMCs in the literature [23], [24], [25], [31], [32], [33].

The Matrix Converter has been proposed by Guygyi-Pelly in 1976 [34], it is a forced commutated converter which uses an array of controlled bidirectional switches as the main power elements to create a variable output voltage system with unrestricted frequency. It does not have any dc-link circuit or components and does not need any large energy storage elements [35].

In [32], a control method with the features of MPC and space vector modulation (SVM) is proposed for a direct matrix converter (DMC). The ability to control different parameters simultaneously is granted by predictive control scheme. The constant switching behavior of the SVM utilized in this method guarantees that a predictive based control method can now be used where the traditional associated problems with input filter sizing, harmonic performance, switching loss and hence thermal management design can now be addressed in a more predictable and systematic way.

Another application of the MPC of a DMC can be found in [33], The method is based on the fictitious DC-link concept, so basically it replaces the DMC with an

equivalent converter, the Indirect Matrix Converter (IMC), in order to separate the control of both input and output stages of the converter.

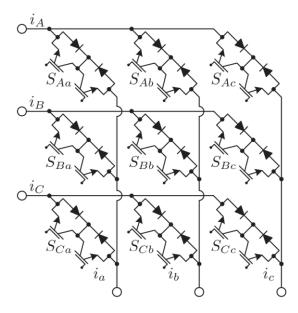


Figure 4: 3×3 Direct Matrix Converter

The results show that both input and output variables are controlled and respond quite well to variations of the references both in transient and steady state and that the operation with a unity power factor is possible and very easy to implement with such control scheme.

In 1989, the indirect matrix converter was introduced by Holtz and Boelkens [34]. The IMC requires separate stages for the voltage and current conversions but without an energy storage element in the intermediate link [36], that is why  $v_{dc}$  is called fictitious dc-link, or virtual dc-link.

Several applications of MPC have been proposed for IMCs in the literature [24], [25], [31], [37].

Researchers in [24] proposed a three-level indirect matrix converter controlled FCS-MPC to achieve load current reference tracking of a RL-load and instantaneous reactive power minimization even under unbalanced supply voltage to demonstrate the robustness of the control system. The control algorithm evaluates 48 possible commutation states of the IMC and chooses specifically the switching states that minimize the cost function. In order to ensure minimum instantaneous reactive power, dedicated cost function has been designed to cater the unbalance voltage supply.

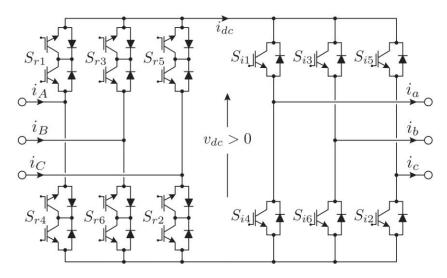


Figure 5:  $3 \times 3$  Indirect Matrix Converter

The experimental results proved an excellent load current reference tracking with low ripple current and the reactive power minimization has been attained by empirical adjustment of the weighting factor. They concluded that the system behavior is highly changeable with the values of the weighting factor in the cost function.

Another application of the MPC of an IMC can be found in motor drives [25] where researchers reduced the torque ripple by exploiting one of the advantages of predictive control, the weighting factor adjustment.

#### IV. Particular applications of Model Predictive Control

Some earlier applications have been mentioned before, the following are fairly recent, particular applications of modern-day MPC.

#### IV.1. Modulated Model Predictive Control

The classic MPC presents a variable switching frequency which could produce high ripple in the controlled waveforms or resonances in the input filter [31], or generate a control signal with a frequency that is higher than the maximum switching frequency of the power switches, which affects the performance of the system.

To overcome this problem, the Modulated MPC (M2PC) has been proposed. This solution maintains all the characteristics of MPC (such as fast transient response ,multi-objective control using only one feedback loop, easy inclusion of non-linearities and constraints of the system, the flexibility to include other system requirements in the

controller) adding the advantages of working at fixed switching frequency, thus improving the quality of the controlled waveforms.

The M2PC utilizes the SVM vector sequence and calculates the duty cycles for each voltage vector based on the minimization of the cost function. Fixed switching frequency is ensured in this case as the sequence of the vectors chosen by the control will be applied within one sampling interval. The difference between a classical MPC and the M2PC is in the application time of the vectors.

The operation at fixed switching frequency is accomplished by emulating space vector modulation using predictive control [31]. By merging space-vector modulation (SVM) within MPC the cost function is used for the optimal selection of the vectors and the respective duty cycles in a sampling instant which are then applied to the converter in the next sampling period [38], [32].

#### IV.1.1. Applications of Modulated Model Predictive Control

In [32], researches investigate the use of M2PC to control a direct matrix converter. The block diagram of their work is presented in the following figure.

A reference current is imposed upon the system and the controller is designed using the system model so that the load current tracks the reference. The measured currents are then used to predict the value of current at (k+1). The reference and predicted currents are then used to calculate the cost function which in turn is used to derive the duty cycles for the selected voltage vectors [32].

The study carried a comparison between M2PC, conventional MPC and the Proportional-Integral (PI) control. The control strategies were applied to a matrix converter feeding an RL load. The results show that the PI controller generates the lowest value of the THD of the output current but it has considerably the highest rise-time. Both predictive control schemes have better transient performance, the rise-time of the M2PC is a little slower but has better quality of the load current.

In [31], a M2PC for an IMC has been proposed. The main objective of this paper is to operate with a unity power factor using two different methods. The control scheme is shown in the following figure.

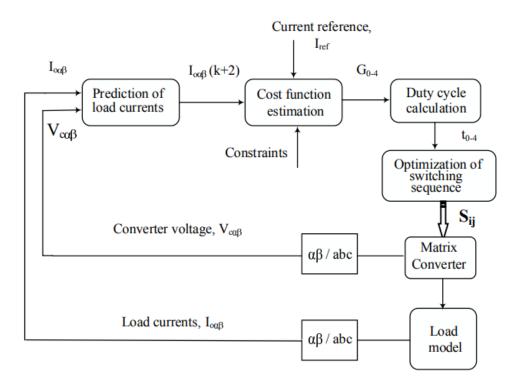


Figure 6: Control block diagram for M2PC strategy for a DMC [32]

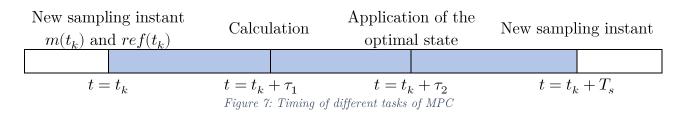
At every sampling instant  $T_s$ , two active vectors are selected for the rectifier and two active vectors and one zero vector for the inverter as well as their respective duty cycles. The switching sequence of the two selected vectors of each side is predefined [31]. And in each method, the switching sequence that minimizes the respective cost function is selected.

Both methods offer a highly satisfying tracking of the load current to its respective reference, with almost sinusoidal waveforms and a very low THD.

# IV.2. Model Predictive Control with extended prediction horizon

MPC with extended prediction horizon is mostly used for delay compensation [3]. This delay can be caused by measurement instruments or estimators.

Another cause of the delay can also be the future reference, because it is compared to the predicted value at the cost function after calculating the predicted values of the controlled variables, which in some cases, requires significant computational time. These calculations cause the reference given to the algorithm (or microprocessor) to be different from the actual reference at that instant.



#### Where:

- $\tau_1$  denotes the time required to acquire a new reference and measurments;
- $au_2$  denotes the time required to calculate the predicted values of the controlled variables for each possible switching state.

Let  $\epsilon$  be an instant between  $\tau_1$  and  $\tau_2$ . If the reference varies with time, it is clear that  $ref\ \epsilon\ \neq ref\ t_k$ , especially when the sampling frequency is not much higher than the frequency of the reference. This error can deteriorate the performance of the system if not considered in the design of the controller

A simple solution to compensate this delay is to take into account the calculation and/or measurement time and apply the selected switching state at the proper sampling instant [39]. For that, an extended prediction horizon is needed and the tasks shown in Figure 7 are modified as follows [32]:

- 1. Measure and/or estimate controlled variables,  $c_k$ ;
- 2. Calculation of the predicted variables for each valid switching state using the mathematical model of the system,  $c_{k+1}$ ;
- 3. Evaluation of the cost function and choosing an optimal actuation  $x_{opt}$ ;
- 4. Redo step 2 while replacing the measured value by  $c_{k+1}$  that corresponds to  $x_{opt}$
- 5. Calculation of the predicted variables for each valid switching state,  $c_{k+2}$
- 6. Evaluation of the cost function for each valid switching state;
- 7. Selection and application of the optimal state that minimizes the cost function.

It is obvious that this solution requires heavy computational efforts, the computational complexity grows exponentially with the length of the prediction horizon and the number of controlled variables.

Another approach is adjust (or estimate) the reference [40]. For sinusoidal references and large sampling times, the use of extrapolation methods for the reference can compensate the delay in the reference tracking in predictive control schemes [3].

A possible solution is to calculate the one-step-ahead prediction using the actual current reference in the nth-order formula of the Lagrange extrapolation [41] given by:

$$r[k+1] = \sum_{j=0}^{n} -1^{n-1} {n+1 \brack j} r[k+1-j]$$
 (I.5)

Where  ${n+1 \brack j}$  denotes the combinatorial operator. This method requires the storage of the most recent n values.

#### IV.2.1. Applications of MPC with extended prediction horizon

In [42], researchers used the first solution to reduce torque ripples of an induction machine driven by a two-level voltage-source inverter (2LVSI). They adopted the Model Predictive Torque Control (MPTC) with an extended prediction horizon.

According to their simulation results, by extending the prediction horizon, important drive quality indices, such as the torque ripple, and the total harmonic distortion (THD) of the stator currents are reduced and the controller is able to improve its closed-loop performance, both under steady state and transient state operating conditions, while enhancing its robustness and stability.

In [3], the authors calculated the future references using the extrapolation (I.5). Where they applied the Model Predictive Current Control (MPCC) to a 2LVSI feeding a resistive-inductive (RL) load. The results show that the estimation of future references achieved an improvement in the waveform of the output current.

These simple compensation methods allow inclusion of the delay in the predictive control schemes and avoid the appearance of large ripples in the controlled variable.

#### IV.3. Non-linear Model Predictive Control

Many systems are inherently nonlinear. The inherent nonlinearity, together with higher product quality specifications and tighter environmental regulations require to operate systems over a wide range of operating conditions and often near the boundary of the admissible region [21]. Under these conditions linear models are often not sufficient to describe the process dynamics adequately and nonlinear models must be used [43].

While linear model predictive control is popular since the 70s of the past century, only since the 90s there is a steadily increasing interest from control theoreticians as well as control practitioners in nonlinear model predictive control (NMPC) [21]. Nonlinear

predictive control is the extension of the well-established linear predictive control to the nonlinear world.

Whereas linear MPC refers to a family of MPC schemes in which linear models are used to predict the system dynamics, NMPC refers to MPC schemes that are based on nonlinear models and/or consider general nonlinear constraints. Though, the working principle is pretty much the same. In NMPC the input applied to the system is usually given by the solution of an optimization problem [22] *i.e.*, a cost function. Nonlinear continuous time systems are described by the following nonlinear differential equation:

$$x \ t = f(x \ t \ , u \ t \ ), \qquad x \ 0 = x_0$$
 (I.6)

Where  $x \ t \in \mathbb{R}^n$  and  $u \ t \in \mathbb{R}^m$  denote the vector of states and inputs, respectively. u and x are often given by box constraints of the form [21]:

$$\begin{split} U &\coloneqq \{u \in \mathbb{R}^m \mid u_{min} \leq u \leq u_{max}\} \\ X &\coloneqq \{u \in \mathbb{R}^n \mid x_{min} \leq x \leq x_{max}\} \end{split} \tag{I.7}$$

Where  $u_{min}$ ,  $u_{max}$ ,  $x_{min}$  and  $x_{max}$  are usually finite constant vectors.

#### IV.3.1. Stability of Non-linear Model Predictive Control

According to the authors of [21] and [22], the ideal NMPC strategy achieves closed-loop stability independent of the choice of the parameters and, if possible, approximates the infinite horizon NMPC scheme as good as possible ( $N_p$  and  $N_m$  in (I.3) are set to  $\infty$ ), which is the first approach to achieve stability. However, normally the solution of a nonlinear infinite horizon optimal control problem cannot be calculated, or at least not sufficiently fast.

Given that infinite horizon NMPC schemes are impractical, finite prediction and control horizons are considered. One approach to ensure stability in finite horizon NMPC, otherwise known as FCS-NMPC, is to add equality or inequality constraints and suitable additional penalty terms in the cost function. These are considered as hard constraints, also known as stability constraints [21]. This implies explicitly that the state  $x\ t$  should obey to the scope statement.

Another approach is the terminal constraint or zero terminal equality constraint [22], the constraint is expressed as:

$$x\left(t + \frac{N_P}{T_s}\right) = 0\tag{I.8}$$

Where  $T_s$  is the time step, or sampling period. This allows the cost function to be qualified as a Lyapunov function [16], [44].

### IV.3.2. Applications of non-linear Model Predictive Control

Areas with the largest number of reported NMPC applications include chemicals, polymers, and air and gas processing [9].

The authors of [43] proposed a model predictive online optimization scheme for the engine torque control problem. The control-oriented model is based on the intake air charging dynamics and torque generation model which are derived from the mean value model. In order to reduce the tracking error induce by the insufficient accurate predictive model, an embedded integrator about the tracking error is designed. Then, the online optimization algorithm namely continuation and generalized minimum residual method (C/GMRES) is adopted to solve the nonlinear optimal problem.

The utilization of dynamic driving test system has been progressively popular in the automotive community in both research and industrial applications [44]. Test system platforms with various mechanical structures have been intended to target specific applications and markets. The driver sensations are replicated by these platforms [45], while the mechanical imperatives must be fulfilled to keep away from incidents [46], which causes a highly nonlinearity in the system.

A survey that was published in the year 2003 revealed that the number of applications of linear MPC largely exceed the number of applications of NMPC [9], this reflects on practical problems encountered in the latter control scheme, which are considerably more challenging than those associated with linear MPC, and computational complexity of NMPC algorithms [9], [21].

#### V. Conclusion

Four decades ago, since the Dynamic Matrix Control was first introduced [10], the research community showed interest in MPC [7]. Since then, MPC technology has progressed steadily and its scope of application expanded from process industry, where time constants are long enough to perform all the required calculations [9], all the way to power electronics and motor drives applications, where time constants are exponentially shorter [3].

Current generation MPC technology offers significant new capabilities thanks to the technological advancement of microprocessors, but several limitations still remain. For instance, the delay caused by measurements and calculations time. Nevertheless, the delay compensation is still a subject of research [39], [42], [47].

MPC covers a wide family of control schemes. This thesis presents an overview of different predictive control methods, alongside with some applications.

The general philosophy underlying predictive control is that by embedding "process knowledge" into the controller, improved performance can be attained. MPC now has a broad range of applications thanks to its advantages:

- very simple and intuitive concepts;
- multivariable case can be easily considered;
- easy inclusion of constraints and non-linearities;
- robustness.

#### References

- [1] J. Holtz, "Pulsewidth modulation for electronic power conversion," *Proceedings of the IEEE*, vol. 82, no. 8, pp. 1194–1214, 1994.
- [2] Q. Lei, F. Z. Peng, and B. Ge, "Pulse-width-amplitude-modulated voltage-fed quasi-Z-source direct matrix converter with maximum constant boost," 2012, pp. 641–646.
- [3] J. Rodriguez and P. Cortes, *Predictive control of power converters and electrical drives*, vol. 40. John Wiley & Sons, 2012.
- [4] A. M. Dadu, S. Mekhilef, and T. K. Soon, "Lyapunov model predictive control to optimise computational burden, reference tracking and THD of three-phase four-leg inverter," *IET Power Electronics*, vol. 12, no. 5, pp. 1061–1070, 2019.
- [5] M. Abdelrahem, Z. Zhang, R. Kennel, H. Eldeeb, and C. Hackl, "Simple and robust direct-model predictive current control technique for pmsgs in variable-speed wind turbines," 2017, pp. 1–6.
- [6] M. Abdelrahem, C. Hackl, R. Kennel, and J. Rodriguez, "Sensorless Predictive Speed Control of Permanent-Magnet Synchronous Generators in Wind Turbine Applications," 2019, pp. 1–8.
- [7] J. H. Lee, "Model predictive control: Review of the three decades of development," International Journal of Control, Automation and Systems, vol. 9, no. 3, p. 415, 2011.
- [8] C. E. Garcia, D. M. Prett, and M. Morari, "Model predictive control: theory and practice—a survey," *Automatica*, vol. 25, no. 3, pp. 335–348, 1989.
- [9] S. J. Qin and T. A. Badgwell, "A survey of industrial model predictive control technology," *Control engineering practice*, vol. 11, no. 7, pp. 733–764, 2003.
- [10] C. R. Cutler and B. L. Ramaker, "Dynamic matrix control?? A computer control algorithm," 1980, no. 17, p. 72.
- [11] J. Richalet, A. Rault, J. Testud, and J. Papon, "Model predictive heuristic control," *Automatica (Journal of IFAC)*, vol. 14, no. 5, pp. 413–428, 1978.

- [12] T. M. Stout and T. J. Williams, "Pioneering work in the field of computer process control," *IEEE Annals of the History of Computing*, vol. 17, no. 1, pp. 6–18, 1995.
- [13] J. B. Froisy, "Model predictive control: Past, present and future," *Isa Transactions*, vol. 33, no. 3, pp. 235–243, 1994.
- [14] D. W. Clarke, C. Mohtadi, and P. Tuffs, "Generalized predictive control—Part I. The basic algorithm," *Automatica*, vol. 23, no. 2, pp. 137–148, 1987.
- [15] D. W. Clarke, C. Mohtadi, and P. Tuffs, "Generalized predictive control—part II extensions and interpretations," *Automatica*, vol. 23, no. 2, pp. 149–160, 1987.
- [16] A. Bemporad and M. Morari, "Robust model predictive control: A survey," in *Robustness in identification and control*, Springer, 1999, pp. 207–226.
- [17] E. G. Gilbert and K. T. Tan, "Linear systems with state and control constraints: The theory and application of maximal output admissible sets," *IEEE Transactions on Automatic control*, vol. 36, no. 9, pp. 1008–1020, 1991.
- [18] S. a Keerthi and E. G. Gilbert, "Optimal infinite-horizon feedback laws for a general class of constrained discrete-time systems: Stability and moving-horizon approximations," *Journal of optimization theory and applications*, vol. 57, no. 2, pp. 265–293, 1988.
- [19] D. Chmielewski and V. Manousiouthakis, "On constrained infinite-time linear quadratic optimal control," *Systems & Control Letters*, vol. 29, no. 3, pp. 121–129, 1996.
- [20] M. Sznaier and M. J. Damborg, "Heuristically enhanced feedback control of constrained discrete-time linear systems," *Automatica*, vol. 26, no. 3, pp. 521–532, 1990.
- [21] F. Allgower, R. Findeisen, and Z. K. Nagy, "Nonlinear model predictive control: From theory to application," *Journal-Chinese Institute Of Chemical Engineers*, vol. 35, no. 3, pp. 299–316, 2004.
- [22] J. B. Rawlings, E. S. Meadows, and K. R. Muske, "Nonlinear model predictive control: A tutorial and survey," *IFAC Proceedings Volumes*, vol. 27, no. 2, pp. 185–197, 1994.

- [23] S. Vazquez, J. Rodriguez, M. Rivera, L. G. Franquelo, and M. Norambuena, "Model predictive control for power converters and drives: Advances and trends," *IEEE Transactions on Industrial Electronics*, vol. 64, no. 2, pp. 935–947, 2016.
- [24] H. M. Basri, K. Lias, and S. Mekhilef, "Digital Predictive Current Control Fed by Three-Level Indirect Matrix Converter under Unbalanced Power Supply Condition."
- [25] M. Uddin, S. Mekhilef, M. Rivera, and J. Rodriguez, "Predictive indirect matrix converter fed torque ripple minimization with weighting factor optimization," 2014, pp. 3574–3581.
- [26] M. P. Akter, S. Mekhilef, N. M. L. Tan, and H. Akagi, "Modified model predictive control of a bidirectional AC–DC converter based on Lyapunov function for energy storage systems," *IEEE Transactions on Industrial Electronics*, vol. 63, no. 2, pp. 704–715, 2015.
- [27] M. P. Akter, S. Mekhilef, N. M. L. Tan, and H. Akagi, "Model predictive control of bidirectional AC-DC converter for energy storage system," *Journal of Electrical Engineering & Technology*, vol. 10, no. 1, pp. 165–175, 2015.
- [28] F. Borrelli, A. Bemporad, M. Fodor, and D. Hrovat, "An MPC/hybrid system approach to traction control," *IEEE Transactions on Control Systems Technology*, vol. 14, no. 3, pp. 541–552, 2006.
- [29] F. Wang, Z. Chen, P. Stolze, R. Kennel, M. Trincado, and J. Rodriguez, "A comprehensive study of direct torque control (DTC) and predictive torque control (PTC) for high performance electrical drives," *EPE Journal*, vol. 25, no. 1, pp. 12–21, 2015.
- [30] P. Cortés, M. P. Kazmierkowski, R. M. Kennel, D. E. Quevedo, and J. Rodríguez, "Predictive control in power electronics and drives," *IEEE Transactions on industrial electronics*, vol. 55, no. 12, pp. 4312–4324, 2008.
- [31] M. Rivera, U. Nasir, L. Tarisciotti, P. Wheeler, T. Dragicevic, and F. Blaabjerg, "Predictive control strategies for an indirect matrix converter operating at fixed switching frequency," 2017, pp. 1–6.
- [32] M. Vijayagopal, P. Zanchetta, L. Empringham, L. De Lillo, L. Tarisciotti, and P. Wheeler, "Control of a direct matrix converter with modulated model-predictive

- control," *IEEE Transactions on Industry Applications*, vol. 53, no. 3, pp. 2342–2349, 2017.
- [33] M. Rivera, L. Tarisciotti, P. Wheeler, and S. Bayhan, "Indirect predictive control strategy with fixed switching frequency for a direct matrix converter," 2017, pp. 7332—7337.
- [34] A. Benachour, "Commande sans capteur basée sur DTC d'une machine asynchrone alimentée par Convertisseur Matriciel," PhD, ENP, Algiers, 2017.
- [35] P. W. Wheeler, J. Rodriguez, J. C. Clare, L. Empringham, and A. Weinstein, "Matrix converters: A technology review," *IEEE Transactions on industrial electronics*, vol. 49, no. 2, pp. 276–288, 2002.
- [36] J. W. Kolar, T. Friedli, J. Rodriguez, and P. W. Wheeler, "Review of three-phase PWM AC–AC converter topologies," *IEEE Transactions on Industrial Electronics*, vol. 58, no. 11, pp. 4988–5006, 2011.
- [37] S. M. Uddin, S. Mekhilef, M. Rivera, and J. Rodriguez, "A FCS-MPC of an induction motor fed by indirect matrix converter with unity power factor control," 2013, pp. 1769–1774.
- [38] B. Majmunović, T. Dragičević, and F. Blaabjerg, "Multi objective modulated model predictive control of stand-alone voltage source converters," *IEEE Journal of Emerging and Selected Topics in Power Electronics*, 2019.
- [39] F. Wang, Z. Zhang, R. Kennel, and J. Rodríguez, "Model predictive torque control with an extended prediction horizon for electrical drive systems," *International Journal of Control*, vol. 88, no. 7, pp. 1379–1388, 2015.
- [40] C. Zheng, T. Dragičević, B. Majmunović, and F. Blaabjerg, "Constrained Modulated Model-Predictive Control of an LC-Filtered Voltage-Source Converter," *IEEE Transactions on Power Electronics*, vol. 35, no. 2, pp. 1967–1977, 2019.
- [41] A. Dekka, B. Wu, V. Yaramasu, R. L. Fuentes, and N. R. Zargari, "Model predictive control of high-power modular multilevel converters—An overview," *IEEE Journal of Emerging and Selected Topics in Power Electronics*, vol. 7, no. 1, pp. 168–183, 2018.

- [42] I. Alevras, P. Karamanakos, S. Manias, and R. Kennel, "Variable switching point predictive torque control with extended prediction horizon," 2015, pp. 2352–2357.
- [43] M. Kang and T. Shen, "Nonlinear model predictive torque control for IC engines," 2014, pp. 804–809.
- [44] D. Hrovat, S. Di Cairano, H. E. Tseng, and I. V. Kolmanovsky, "The development of model predictive control in automotive industry: A survey," 2012, pp. 295–302.
- [45] L. Fridman *et al.*, "MIT advanced vehicle technology study: Large-scale naturalistic driving study of driver behavior and interaction with automation," *IEEE Access*, vol. 7, pp. 102021–102038, 2019.
- [46] Y. Chen, "Algorithms and Applications for Nonlinear Model Predictive Control with Long Prediction Horizon," 2018.
- [47] Y. Han, C. Gong, L. Yan, H. Wen, Y. Wang, and K. Shen, "Multi-Objective Finite Control Set Model Predictive Control Using Novel Delay Compensation Technique for PMSM," *IEEE Transactions on Power Electronics*, 2020.