Available online at www.sciencedirect.com

ScienceDirect

Energy Procedia 139 (2017) 173–179

Energy Procedia

www.elsevier.com/locate/procedia

International Conference On Materials And Energy 2015, ICOME 15, 19-22 May 2015, Tetouan, Morocco, and the International Conference On Materials And Energy 2016, ICOME 16, 17-20 May 2016, La Rochelle, France

Lattice Boltzmann application for a viscoplastic fluid flow and heat transfer into cubic enclosures

Abdelkader BOUTRA[a,b,]*, Karim RAGUI[a], Nabila LABSI[a], Youb Khaled BENKAHLA[a], Rachid BENNACER[c]

[a] *Laboratory of Transport Phenomena, USTHB, Algiers, Algeria*
[b] *Preparatory School of Science and Technology, Algiers, Algeria.*
[c] *LMT - ENS Cachan - CNRS - Paris Saclay University, 94230 Cachan, France*

Abstract

The present work deals with a Tri-dimensional natural convection into a cubical enclosure, completely filled with a yield fluid which obeying the Bingham rheological model. A thermal gradient is governed by a uniform temperature imposed on the right and the left walls, when the other surfaces are kept insulated. To solve the governing equations, a numerical code based on the Lattice Boltzmann method is used. The latter has been validated after comparison between the present results and those of the literature. Regarding the Bingham number effect on heat transfer rate inside the enclosure, the convection phenomenon is analyzed through the isotherm plots and its temperature profiles with special attention to the local Nusselt number of the active walls.

Peer-review under responsibility of the scientific committee of ICOME 2015 and ICOME 2016.

Keywords: Numerical investigation; natural convection; cubic enclosure; Bingham fluid; Lattice Boltzmann method

* Corresponding author. Tel.: +213-561-765-333; fax: +213-214-167-77.
E-mail address: aeknad@yahoo.fr

Peer-review under responsibility of the scientific committee of ICOME 2015 and ICOME 2016
10.1016/j.egypro.2017.11.192

1. Introduction

Over the last four decades, natural convection of Non Newtonian fluids has received a considerable attention by many researchers [1-5]. This interest stems from its importance and its wide range of applications; such in crude oil tanks, channel heat sinks and compact heat exchangers, to name but a few [6,7].

As shown the literature, the two dimensional natural convection of such complex fluids into many configurations becomes the classical research problem extensively investigated, cited then; Syrjälä [8], Chen et *al*.[9], Frey et *al*.[10], and Capobianchi et *al*.[11] as well, in the main idea to describe the flow and heat transfer characteristics of such fluids into these geometries. Mitsuoulis and Zisis [12] performed a numerical investigation of a yield fluid obeying the rheological model of Bingham into a square enclosure. A Few years later, Turan et *al*. [13, 14] proved that heat transfer of this Viscoplastic fluid can be numerically predicted. In their papers, the mean Nusselt Number of a square enclosure, completely filled with a Bingham fluid, is found as a function of the governing parameters such the Rayleigh, the Prandtl, and the Bingham numbers.

Even though there have been these numerous investigations in 2D, under many configurations and boundary conditions, relatively few studies were documented by taking into account the third dimension. As such, this paper presents a comprehensive numerical investigation of a Bingham' natural convection into a cubical enclosure with an horizontal temperature gradient; generated by its left and right walls. The numerical work is developed using the Lattice Boltzmann method side by side with the finite difference one [15, 16].

Nomenclature

Bn	Bingham number, $= \tau_0 H / \mu V_0$	U, V, W	Dimensionless velocity components
C_p	Specific heat transfer, J $kg^{-1}K^{-1}$	**Greek symbols**	
$I\gamma$	Second invariant of strain rate tensor	β	Thermal expansion coefficient (K^{-1})
k	Thermal conductivity, W m^{-1} K^{-1}	γ	Strain rate, s^{-1}
m	Exponential growth parameter	θ	Dimensionless temperature
M	Reduced exponential growth parameter	μ	Dynamic viscosity kg m^{-1} s^{-1}
Nu	Wall Nusselt number	η	Effective viscosity, kg m^{-1} s^{-1}
p	Pressure, Pa	ρ	Fluid density, kg m^{-3}
P	Dimensionless Pressure	τ	Shear stress, Pa
Pr	Prandtl number, $= C_p\ \mu/k$	τ_0	Yield stress, Pa
Ra	Rayleigh number, $= g\ \beta\ \rho^2\ \Delta T\ H^3\ C_p/\mu\ k$	**Subscripts**	
T	Temperature, K	c	Cold
u, v, w	Velocity components, m s^{-1}	h	Hot

2. Problem statement and Mathematical formulation

The studied configuration, shown in Fig. 1, consists of a cubical enclosure completely filled with a Viscoplastic Bingham fluid. The left and the right vertical walls are maintained at constant, but different, temperatures when the other walls are assumed to be insulated. The thermo-physical properties of the investigated fluid are assumed to be constant, except for the viscosity which depends on shear rate [17], and the density variation, in the buoyancy term, which follows the Boussinesq approximation [18].

Regarding the Lattice Boltzmann equation for the Bingham fluid, we considered a nineteen-velocity model of a three-dimensional lattice so-called D3Q19 model [19] (*see for instance Fig.1(b)*).

For the adopted dynamic model, two successive steps are taken into account, such the propagation of particles from nodes to their neighbors and the collision between the various velocities, noted as (c_i), at each lattice node. The particles distribution equation is expressed as follows:

$$\frac{\partial \vec{f}}{\partial t} + \vec{c}\,\nabla\vec{f} = \left(\frac{\partial \vec{f}}{\partial t}\right)_{scat} \qquad (1)$$

where $f(x,c,t)$ is the distribution function which depends on the particle velocity "c" at a location "x" and a time "t". The right hand side term of Eq. (1) displays the diffusion process; when the new equilibrium distribution is rebuilt after the collision. According to Guo et *al*. [20], this term is proved to be non-linear and there are different methods to treat it.

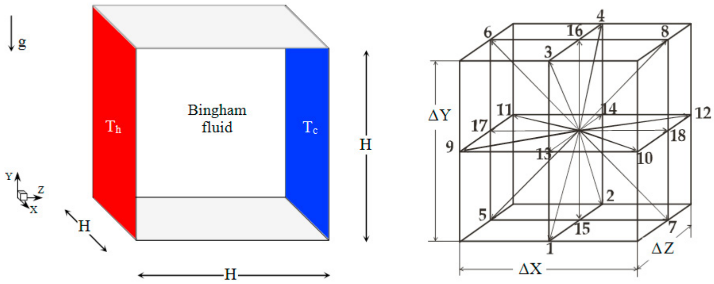


Fig. 1. (a) Geometrical configuration, (b) Velocity model D3Q19.

For the D3Q19 model, the fluid state can be defined as particles populations vector f_i, (i = 0, 1, 2,…, 18), depending on the location x and the time t. Then, the discrete distribution equation may resume as:

$$f_i(x + c_i\Delta t, t + \Delta t) = f_i(x, t) + (\Omega f)_i \quad (2)$$

where f_i is the space vector based on the discrete velocity set and Ω is the collision operator. Elementary discrete velocity sets will be created then from the set of 18 vectors pointing from the origin to the above neighbors and the zero vector (0,0,0).

Noted that the space vector f_i is constructed using the moment of the latter. The relationship between two spaces is defined by means of the below equation, when the coefficient a_{ij} is calculated using the particle velocity c_i [21].

$$m_j = \sum_i a_{ij} f_i \quad (3)$$

About the energy equation, and because of the non-existence of non-linearity, the finite difference scheme is found more required than the LBE-scheme. The relation between the temperature and flow fields is found to be as far as the force is in y-direction, which arises with the temperature gradient, is introduced. The latter is thus, used in the y-velocity calculations, as shown in the advection term of the energy equation cited below:

$$\frac{\partial T}{\partial t} + \vec{V}\nabla T = k\Delta T \quad (4)$$

At the end, it is worth to denote that the mean Nusselt number computed along the active walls is obtained through the following expressions:

$$|Nu_{Hot}| = \int_0^H \int_0^H (\partial T/\partial x)_{x=0 \text{ or } x=H}\, dydz \quad (5)$$

3. Numerical validation

The performance of the using computer code via a 3D natural convection problem is established by comparing our predictions with other numerical results, namely those of Fusegi et *al*. [22] and Frederick et *al*. [23]. By taking into account the same hypotheses, Table 2 demonstrates a comparison of the mean Nusselt number computed inside the Air cubical enclosure. As we can see, the present results and those of references [22, 23] are in excellent agreement, with a maximum discrepancy of about 2%.

To validate the code via a Non-Newtonian fluid, the 2D predictions of Turan et *al*. [13] are taken into account as shown in Fig. 2. Once again, a great similarity between the two results is verified, with a maximum discrepancy of about 1.8%. In the light of the above validation tests, the present numerical scheme can be expected to yield very accurate velocity, temperature and Nusselt number predictions for the considered problem.

In order to determine a proper grid for the numerical simulations, a grid independence study is conducted for the natural convection into the cubical enclosure shown in Fig. 1. Several mesh distributions ranging from 41^3 to 101^3 were tested. The mean Nusselt number of the cube for the above uniform grids is presented in Fig. 3. It is observed that a 71^3 uniform grid is adequate for a grid independent solution. However, a fine structured mesh of 81^3 was used to avoid round-off error for all other calculations in this investigation.

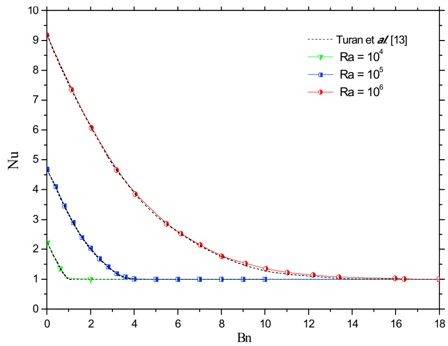


Fig. 2. Mean Nusselt number as a function of Bingham and Rayleigh numbers, Pr = 7.

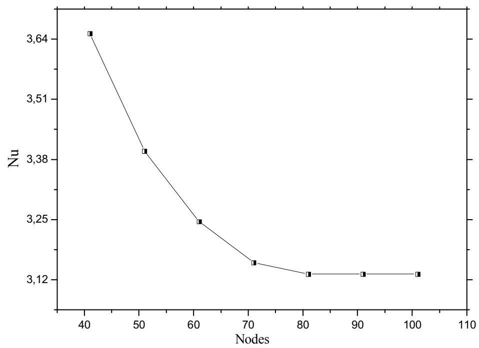


Fig. 3. Average Nusselt number of the cubical enclosure for various uniform grids. Bn = 4, Ra = 10^6

Table 1. Average Nusselt number obtained with the computer code and references [22-23], Pr = 0.71.

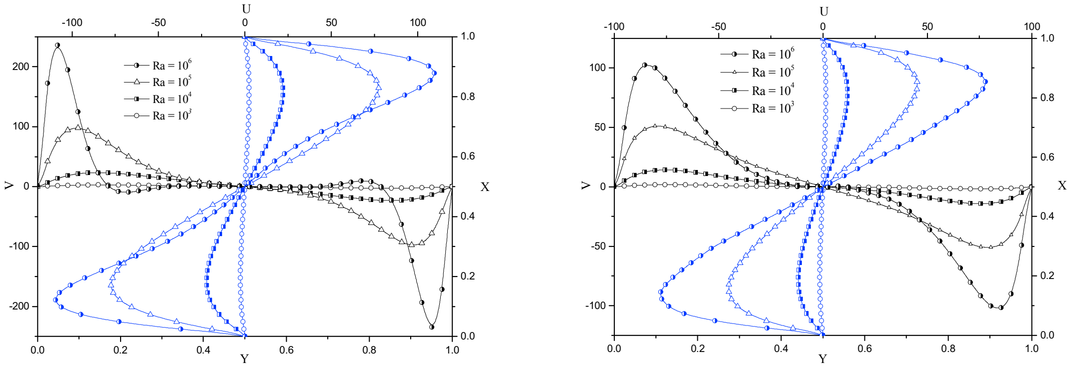
Ra	Mean Nusselt Number		
	Fusegi et *al*. [22]	Frederick et *al*. [23]	Present Predictions
10^3	1.085	1.071	1.071
10^4	2.100	2.057	2.062
10^5	4.361	4.353	4.367
10^6	8.770	8.740	8.761

4. Results and discussion

The main parameter that may govern the convection phenomenon is the Rayleigh number, as its presents the ratio of buoyancy and viscous forces while maintaining proportionally to the applied temperature gradient. Thus, Fig. 4 illustrates the evolution of both; the horizontal and vertical velocity' components with Rayleigh number and that, for two different values of Bingham number such as 0 and 0.5, respectively. The Prandtl number is fixed at 10. According to this figure, the velocities' amplitude near the vertical and horizontal walls is an increasing function of the increase Rayleigh. The existence of a stagnant fluid mass (predicted, but not presented) near the center of the enclosure may explain the velocity zero value. It should be noted that the velocity' profiles in the Newtonian fluid case are more important than those of the Bingham one, which is due to the fact that Newtonian fluid is less viscous.

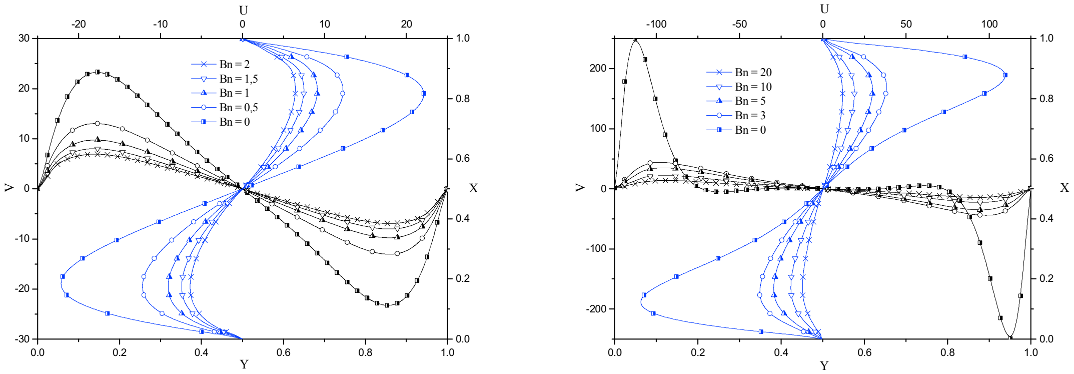
According to Fig. 5, which displays the effect of the Bingham number on the velocity profiles (for two different values of Rayleigh number), the increase in the Bingham number increases the apparent viscosity of the fluid what affects the ability of the fluid to flow and then, decreases the velocities into the enclosure.

As far as the thermal part is considered, Fig.6 illustrates the temperature profiles measured in the mid-plane (X, 0.5, 0.5) of the cube, for various values of the Bingham number. A thin boundary is observed nearer the active walls, which increases by increasing the Bingham number. For Bn = 20, the temperature profile decreases linearly;



(Bn = 0) (Bn = 0.5)

Fig. 4. Evolution of V (0,y,1/2) and U (x,0,1/2) with Rayleigh number, Pr = 10.



(Ra= 10^4) (Ra= 10^6)

Fig. 5. Evolution of V (0,y,1/2) and U (x,0,1/2) with Bingham number, Pr = 10.

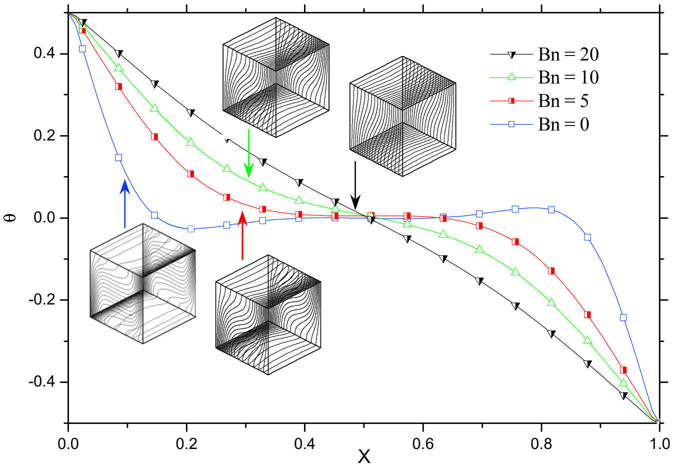


Fig. 6. Adimensional temperature profiles, θ (X, 0.5, 0.5), for various Bingham numbers, Ra = 10^5, Pr = 10.

As the heat transfer is dominated by the conduction mode. The thickness of the thermal boundary layer is then, equal to the cavity length. With Bn < 10, a convective mode is dominated, the latter is very pronounced with Bn = 0 in particular, which corresponds to the Newtonian fluid case. As Bn = 5, Fig. 7 shows the evolution of temperature profiles measured in the mid-plane of the third direction (Z = 0.5), for different values of Y. For the horizontal planes corresponding to Y < 0.5, we denote that the lower half of the enclosure is mainly occupied by the cold fluid. Conversely, the hot fluid holds the majority of the upper part.

The evolution of the local Nusselt number along the left hot wall, for various values of the Bingham number, is presented in Fig. 8. The increase in Bingham number decreases the Nusselt values except for Bn = 10 and 20 for which, reversal phenomenon is observed when Y > 0.6. In the case of Bn = 0 and 5, the Nusselt value gets lower as a function of (Y) from a specific altitude (Y > 0.1 and Y > 0.15 respectively) until it reaches its minimum value in the upper portion the hot wall due to the low temperature gradients. This behavior is less pronounced for important values of Bingham number such Bn = 10 and 20.

5. Conclusion

Laminar natural convection of a Viscoplastic fluid was investigated through this paper. This flow was taking place inside a cubicle enclosure maintained at a horizontal temperature gradient. The results show that the third direction plays a crucial role on the heat transfer, besides the Bingham number which characterizing the fluid nature. As expected, the Nusselt number is found as a decreasing function of the Bingham number and that for all simulated cases, except for Bn = 10 and 20 for which, a reversal phenomenon is observed with Y > 0.6.

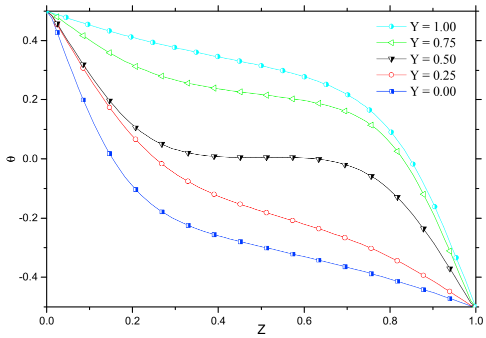


Fig. 7. Adimensional temperature profile evolution, θ (X, Y, 0.5). Ra = 10^5, Pr = 10, Bn = 5.

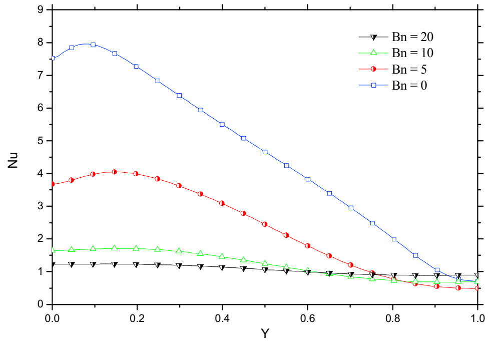


Fig. 8. Effect of Bingham number on the local Nusselt number evaluated along the hot wall. Ra = 10^5.

References

[1] Manglik R.M., Ding J. Laminar flow heat transfer to viscous power law fluids in double-sine ducts, Int. J. Heat Mass Transf. 1997;40: 1379-1390.

[2] Wang C.C., Chen C.K.. Mixed convection boundary layer flow of non-Newtonian fluids along vertical wavy plates, Int. J. Heat Fluid Flow; 2002, 23: 831-839.

[3] Lamsaadi M., Naïmi M., Hasnaoui M.. Natural convection of non-Newtonian power law fluids in a shallow horizontal rectangular cavity uniformly heated from below, Heat Mass Transf. 2005; 41: 239-249.

[4] Pinho F.T., Coelho P.M.. Fully-developed heat transfer in annuli for viscoelastic fluids with viscous dissipation, J. Non-Newtonian Fluid Mech.; 2006, 138: 07-21.

[5] Habibi Matin M., Khan W.A. Laminar natural convection of non-Newtonian powerlaw fluids between concentric circular cylinders, Int. Commun. Heat Mass Transfer 2013, 43: 112-121.

[6] Hartnett J.P., and Kostic M. Heat transfer to Newtonian and non-Newtonian fluids in rectangular ducts", Advances in heat transfer, 1989, 19: 247-356.

[7] Bayazitoglu Y., Paslay P.R., Cernocky P. Laminar Bingham fluid flow between vertical parallel plates, Int. J. Therm. Sci. (2007) 46: 349-357.

[8] Syrjälä S., Further finite element analyses of fully developed laminar flow of power-law non-Newtonian fluid in rectangular ducts: heat transfer predictions, Int. Commun. Heat Mass Transf. 1996, 23(6): 799-807.

[9] Chen Y.L., Cao X.DZhu., K.Q. A gray lattice Boltzmann model for power-law fluid and its application in the study of slip velocity at porous interface, J. Non-Newtonian Fluid Mech. 2009, 159: 130-136.

[10] Frey S., Silveira F.S., Zinani F. Stabilized mixed approximations for inertial viscoplastic fluid flows. Mechanics Research Commun. 2010, 37: 145-152.

[11] Capobianchi M., Wagner D. Heat transfer in laminar flows of extended modified power law fluids in rectangular ducts, Int. J. Heat Mass Transf. 2010, 53: 558-563.

[12] Mitsoulis E., Zisis Th. Flow of Bingham plastics in a lid-driven Square cavity, J. of Non-Newtonian Fluid Mechanics, 2001, 101: 173-180.

[13] Turan O., Chakraborty N., Robert P.J. Laminar natural convection of Bingham fluid in a square enclosure with differentially heater side walls. J Non-Newtonian Fluid Mech 2010,165: 901-913.

[14] Turan O., Chakraborty N., Robert P. J. Laminar Rayleigh-Bénard convection of yield stress fluids in a square enclosure, J. Non-Newt. Fluid Mech, 2012,171: 83-96.
[15] Lallemend P., Luo L. Hybrid finite-difference thermal lattice Boltzmann equation, Int. J.Modern Physics. B, 2003,17-1/2 : 41-47.
[16] Mezrhab A., Bouzidi M., Lallemand P. Hybrid lattice Boltzmann finite-difference simulation of convective flows, Computer and Fluids, 2004, 33: 623-641.
[17] Papanastasiou T. C. Flows of materials with yield, J. of Rheology, 1987, 31: 385-404.
[18] Bejan A. Convection heat transfer, John Wiley & Sons, Inc., Hoboken, New jersey, USA, 2004.
[19] d'Humières D., Generalized lattice-Boltzmann equations, in Rarefied Gas Dynamics: The theory and simulations. In: B.D. Shizgal and D.P. Weaver, Editors, AIAA Progress in astro. Aero. 1992,159: 450-458.
[20] Guo Y., Bennacer R., Shen S., Ameziani D.E., Bouzidi M., Simulation of mixed Convection in a slender rectangular cavity with a Lattice Boltzmann Method, Int. J. Nume. Methods Heat Fluid Flow. 2010, 20: 130-148.
[21] Boutra A., Ragui, K., Labsi N., Bennacer R., Benkahla Y.K., Natural Convection Heat Transfer of a Nanofluid into a Cubical Enclosure: Lattice Boltzmann Investigation. Arab J Sci Eng. 2016: 01-14.
[22] Fusegi T., Hyun J.M., Kuwahara K., Farouk B. A numerical study of three-dimensional natural convection in a differentially heated cubical enclosure. Int. J. Heat Mass Transfer. 1991, 34: 1543-1557.
[23] Frederick R.L., Moraga S.G. Three-dimensional natural convection in finned cubical enclosures. Int. J. Heat Fluid Flow. 2007, 28: 289-298.