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Critical Dimension of a Circular Heat and Solute Source for an Optimum Transfer within Square Porous Enclosures

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Abstract

The present work refers to the investigation of natural convection within a partitioned porous enclosure, driven by cooperating thermal and solutal buoyancy forces. The side walls are maintained at a uniform temperature and concentration, lower than that of a circular heat and solute source, which located at the center of the porous square, the rest of the horizontal walls are kept insulated. The physical model for the momentum conservation equation makes use of the Brinkman extension of the classical Darcy equation, the set of coupled equations is solved using the finite volume approach and the SIMPLER algorithm. To account for the impact of the main parameters such the buoyancy ratio; Lewis and porous thermal Rayleigh numbers; as well as the source dimension, heat and mass transfer characteristics are widely inspected and then new powerful correlations are proposed, which predict within ±1% the numerical results. Noted that the validity of the used code was ascertained by comparing our results with experimental data and numerical ones; already available in the literature.

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Keywords: Double-diffusive convection; square porous enclosure; circular source; finite volume approach; powerful correlations

1. Introduction

Double-diffusive natural convection analysis within a porous medium has been the subject of a very intense research activity over the past forty decades reason to the importance of related industrial and technological applications; such as fibrous insulating materials, heat exchangers, catalytic reactor and some modes of assisted oil recuperation [1], to name but a few.

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With both temperature and concentration gradients present to drive the fluid flow, an increased number of transport configurations was possible; with parallel or perpendicular gradients; and the body forces augmenting or opposing [2-8].

In the main idea to predict heat and mass transfer into such configurations, powerful correlations were proposed. Back to 1987, Trevisan and Bejan [9] projected the thermal and solutal transfer, quantified by Sherwood and Nusselt numbers, as function of the thermal Rayleigh and the Lewis numbers as well as the aspect and the buoyancy ratios. In 1990 Lin et *al*. [10] proposed Nusselt and Sherwood correlations as function of the thermal Grashof number and that, for small values of the buoyancy ratio ($|N| < 5$). In 1993, Bennacer [11] suggested a general correlation for mass transfer inside square enclosures, which may used in a wide range of the porous thermal Rayleigh number, the buoyancy ratio and the Lewis number as well. In 2016, Ragui et *al*. [12] proposed general correlations for thermosolutale convection into square enclosures; which including a central bottom source.

Motivated by these works and the practical applications of the double-diffusive convection, this paper will discuss the results of this phenomenon within a porous square; included a heat and solute circular source, mounted at the center, to bring up at the end some powerful correlations that may use in many industrial applications, especially in cooling cylindrical fuel assemblies and predicting pollutants spreading into heat exchangers.

Nomenclature

C	Dimensional mass fraction
d	Circular source' diameter, [m]
D	Mass diffusivity, [$m^2 s^{-1}$]
Da	Darcy number, K/H^2
H	Cavity height, [m]
K	Porous medium permeability, [m^2]
Nu	Mean Nusselt number
p	Pressure, [Pa]
P	Dimensionless pressure, ($p/\rho\ \beta\ \Delta T\ H$)
Sh	Mean Sherwood number
T	Dimensional Temperature, [K]
u, v	Velocity components, [$m\ s^{-1}$]
U, V	Dimensionless velocity components, u (or v)/($\rho\ \beta\ \Delta T\ H)^{1/2}$
x, y	Cartesian coordinates, [m]
X, Y	Dimensionless Cartesian coordinates, x (or y)/H.
Greek letters	
α	Thermal diffusivity, [$m^2 s^{-1}$]
β_T	Thermal expansion coefficient, [K^{-1}]
β_C	Solutal expansion coefficient
ϕ	Dimensionless concentration
θ	Dimensionless temperature
Subscripts	
h	Hot
c	Cold

2. Problem statement & Mathematical formulation

The studied configuration, shown in Fig. 1, consists of a cold (less concentric) side-walls of a porous enclosure, which containing a heat and solute circular source, mounted in its center. The fluid filled the porous medium is assumed to be Newtonian. Its thermophysical properties are presumed to be constant except the density variation, in the buoyancy term, which depends linearly on both the local temperature and concentration.

$$\rho_{(T,C)} = \rho_0 [1 - \beta_T (T - T_0) - \beta_C (C - C_0)] \tag{1}$$

where β_T and β_C are the thermal and solutal expansion coefficients:

$$\beta_T = -\frac{1}{\rho_0}\left(\frac{\partial \rho}{\partial T}\right)_p,\ \beta_C = -\frac{1}{\rho_0}\left(\frac{\partial \rho}{\partial C}\right)_p \tag{2}$$

The solid matrix is supposed to be isotropic, homogeneous and in thermal equilibrium with the fluid. The permeability of the porous medium K is kept uniform, when the porosity ε is about 40%.

The dimensionless conservation equations describing the transport phenomenon inside the porous square can be written as:

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \tag{3}$$

$$\frac{1}{\varepsilon^2}\left(U\frac{\partial U}{\partial X} + V\frac{\partial U}{\partial Y}\right) = -\frac{\partial P}{\partial X} - \frac{Pr}{Da}U + Pr\left(\frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2}\right) \tag{4}$$

$$\frac{1}{\varepsilon^2}\left(U\frac{\partial V}{\partial X}+V\frac{\partial V}{\partial Y}\right)=-\frac{\partial P}{\partial Y}-\frac{Pr}{Da}V+Pr\left(\frac{\partial^2 V}{\partial X^2}+\frac{\partial^2 V}{\partial Y^2}\right)+Ra\,Pr\,(\theta+N\,\phi) \tag{5}$$

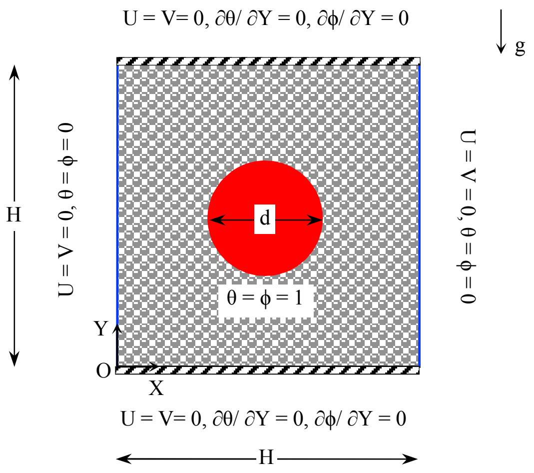


Fig. 1. Simulation domain with its boundary conditions.

Table 1. Average Nusselt and Sherwood numbers obtained with our computer code and those of Hadidi et *al*. Ra = 10^6, Pr = 7, Da = 10^{-4}, Le = 10.

	Hadidi et *al*. [13]		Present Work	
N	Nu	Sh	Nu	Sh
0	2.83	10.25	2.79	10.29
10	3.95	26.30	3.91	26.33
15	4.57	29.75	4.56	29.81

$$U\frac{\partial\theta}{\partial X}+V\frac{\partial\theta}{\partial Y}=\left(\frac{\partial^2\theta}{\partial X^2}+\frac{\partial^2\theta}{\partial Y^2}\right) \tag{6}$$

$$U\frac{\partial\phi}{\partial X}+V\frac{\partial\phi}{\partial Y}=\frac{1}{Le}\left(\frac{\partial^2\phi}{\partial X^2}+\frac{\partial^2\phi}{\partial Y^2}\right) \tag{7}$$

where ε is the porosity, Da is the Darcy number, Le is the Lewis number, N is the buoyancy ratio, Pr and Ra are the Prandtl and the thermal Rayleigh numbers.

The average rate of heat and mass transfer across the side walls are expressed in dimensionless form by the Nusselt and Sherwood numbers such:

$$\left|Nu_{\text{Vertical walls}}\right|=\int_0^1\left(\frac{\partial\theta}{\partial X}\right)\Bigg|_{\text{wall}}dY \quad ; \quad \left|Sh_{\text{Vertical walls}}\right|=\int_0^1\left(\frac{\partial\phi}{\partial X}\right)\Bigg|_{\text{wall}}dY \tag{8}$$

3. Numerical procedure and Validation

The governing conservation equations are discretized in space using the finite volume approach, when the convection-diffusion terms were treated with a Power-Law scheme. The resulting algebraic equations, with the associated boundary conditions, are then solved using the line by line approach. As the momentum equation is formulated in terms of the primitive variables (U, V and P), the iterative procedure includes a pressure correction calculation method, namely SIMPLER [13], to solve the pressure-velocity coupling. Noted that the convergence criterion for temperature, concentration, pressure, and velocity is given as:

$$\frac{\sum_{j=1}^{m}\sum_{i=1}^{n}\left|\xi_{i,j}^{t+1}-\xi_{i,j}^{t}\right|}{\sum_{j=1}^{m}\sum_{i=1}^{n}\left|\xi_{i,j}^{t+1}\right|}\le 10^{-5} \tag{9}$$

where m and n are the numbers of grid points in the x- and y-directions, respectively. ξ is any of the computed field variables, and t is the iteration number.

The performance of the using code via the double-diffusive natural convection problem in a confined porous medium is established by comparing predictions with other numerical results and experimental data, and by verifying the grid independence of the present results. First, the present results are consistent with previous computations, namely those of Hadidi et *al*. [14]. By taking into account the same hypotheses, Table 1 demonstrates a comparison of the mean Nusselt and Sherwood numbers computed with various values of the buoyancy ratio. As we can see, the present results and those of Hadidi et *al*. are in excellent agreement with a maximum discrepancy of about 2%.

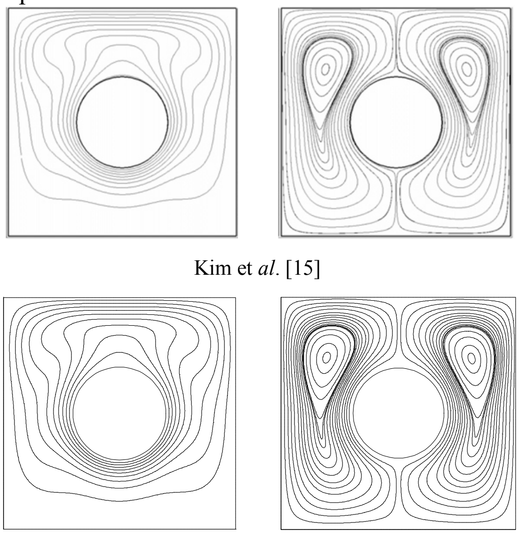


Fig. 2. Comparison between the present results and those reported by Kim et *al*. [15], Pr = 0.71.

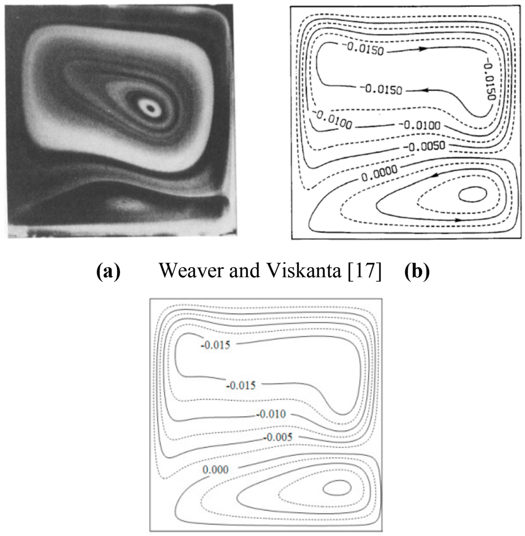


Fig. 3. Ethanol/Nitrogen binary fluid in a square cavity,
(a)- Gr = 1.157 10^6, N = -2.335, Pr = 0.802, Sc = 0.555.
(b)- Gr = 1.121 10^6, N = -2.328, Pr = 0.802, Sc = 0.557.

To ascertain the numerical code validity with the inner circular results, those obtained by Kim et al. [15] for a cold enclosure containing a centered hot cylinder have been selected. Fig. 2 displays the comparison between the numerical Kim et *al*. predictions and the present ones in term of streamlines and Isotherm plots, Table 2 illustrates the obtained mean Nusselt number and both; Kim et al. and Sheikholeslami et *al*. [16], for various Rayleigh values. A great agreement between our results and both works is reported, what validates our code via the circular shape.

Then, to check the numerical code validity with experimental results, those obtained by Weaver and Viskanta [17] for an Ethanol/Nitrogen binary fluid have been selected. Fig. 3 displays the comparison between the experimental data and both, the numerical Weaver and Viskanta predictions and the present ones in term of velocity contours. Once again, the numerical results show a good qualitative concordance with the experimental data and a great agreement with the numerical Weaver and Viskanta predictions.

In order to determine a proper grid for the numerical simulations, a grid independence study is conducted for the convection phenomenon inside the porous squares previously shown in Fig. 1. Several mesh distributions ranging from 161^2 to 401^2 were tested and the mean Sherwood number of the squares, for the above uniform grids, is presented in Fig. 4. It is observed that a 201^2 uniform grid is adequate for a grid independent solution. However, a fine structured mesh of 241^2 is used to avoid round-off error for all other calculations in this investigation.

4. Results & Discussion

The range of parameters that has been examined in this study concerns the cooperating buoyancy forces domain. The value of the buoyancy ratio has been taken from 0 to 30 and that, for various values of the porous thermal Rayleigh number (Ra_T Da), which ranging from 100 to 2000. The Lewis number has been taken between 10 and 300. The source diameter has been taken between 10% and 90% the enclosure length, when both the Prandtl and the Darcy numbers are fixed at 10 and 10^{-3}, respectively.

4.1. *Impact of Lewis Number & buoyancy ratio*

When the buoyancy ratio and the source diameter are fixed at 10 and 0.20, respectively, the results illustrated in Fig. 5 have been obtained for two different values of Lewis number such as 10 and 100, respectively. By presenting the profile of the vertical velocity ratio V/V_{max}, in the mid-plane of the enclosure, side by side with the temperature and the concentration profiles, it is found that the flow driven by buoyancy near the side-walls is due primarily to concentration gradient; as the temperature variation is insignificant, which is not the same near the circular source where; the buoyancy effect is caused by the combined effect of the temperature and the concentration variations.

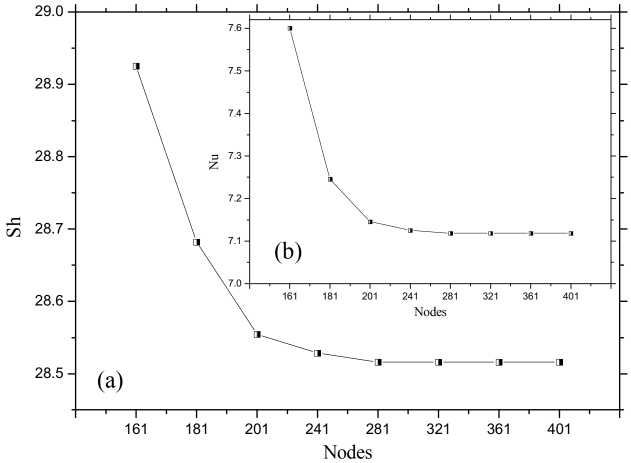


Fig. 4. Mean Sherwood **(a)** and Nusselt numbers **(b)** of the porous enclosure for different uniform grids. Da = 10^{-3}, Le = 10, N = 10, Ra* = 100, d = 0.40.

Table 2. Mean Nusselt number obtained by the present solution and previous works [15, 16], for various Rayleigh numbers, Pr = 0.71.

Ra	Kim et *al.*[15]	Sheikholeslami et *al.* [16]	Present work	Δ(%)
10^3	4.95	4.97	4.97	**0.42**
10^4	5.03	5.09	5.05	**0.40**
10^5	7.71	7.67	7.72	**0.13**

The increase in Lewis number causes the diminution of the thickness of the mass boundary layer near the active walls unlike the temperature one, what enhances accordingly the computed local Sherwood value and decrease the Nusselt one.

When the Lewis number is fixed at 10, Fig.6 displays the impact of the buoyancy ratio on the flow field, side by side with the temperature and the concentration. As for the last one, (*i.e.* Fig.5), and near the side-walls, the flow driven by buoyancy is due primarily to concentration gradient; as the temperature variation still insignificant. Near the circular source, the buoyancy effect is caused by the combined effect of the temperature and the concentration variations.

The increase in the buoyancy ratio makes the thermal and the solutal boundary layers thinner near the active walls, what improves the local transfer characteristics (*i.e. local Nu and Sh*) into these regions and so the overall thermosolutal transfer within the porous enclosure.

4.2. *Impact of the source' diameter*

As far as the source diameter effect is conducted, Fig.7 displays the thermal and solutal transfers as a function of the source geometry which reported by the aspect ratio (d/H) and that, for various values of the Lewis and buoyancy ratio, (Ra* = 100).

For a given value of Le (or N as well), a further increase in the aspect ratio to 0.80 causes a significant improvement in thermosolutal convection, as the latter controls this phenomenon within the porous square. Going far with its value, the use of an aspect ratio greater than 0.80 die out the impact of Lewis and buoyancy ratio on the mean heat transfer as the conductive regime will take control.

After this critical value, (*i.e. d/H = 0.80*), the Sherwood number decreases a little before starting another increase. This reduction may relate to the fluid mass which is very low and that, raison of the enhancement of the source diameter. The increase in the Sherwood number once again can be related to the pure diffusion regime, which can be reached by using an important aspect ratio such as 0.90.

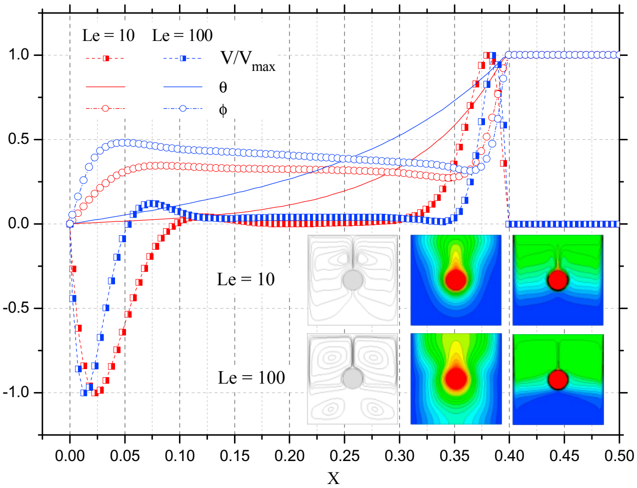


Fig. 5. V/V_{max}, θ and ϕ profiles at the mid-section of the enclosure for various values of Lewis number. Da = 10^{-3}, N = 10, Ra* = 100, d = 0.20.

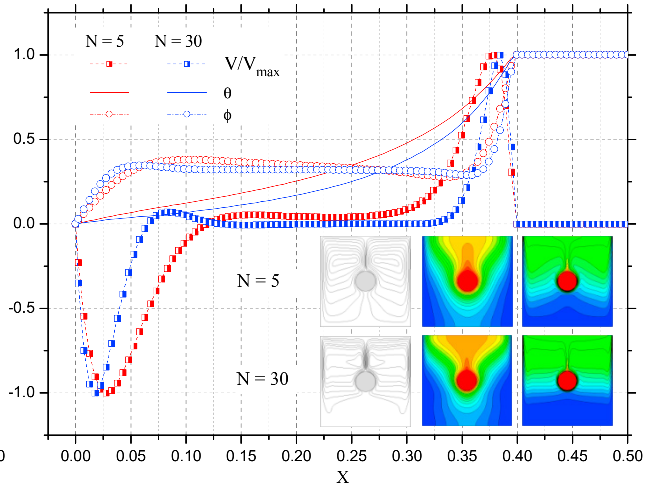
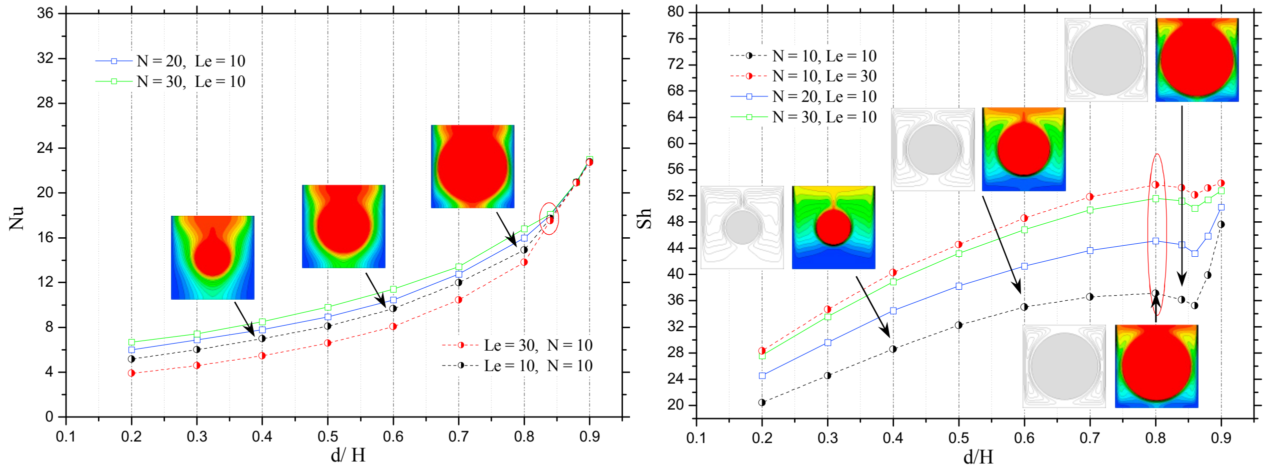


Fig.6. V/V_{max}, θ and ϕ profiles at the mid-section of the enclosure for various values of buoyancy ratio. Da = 10^{-3}, Le = 10, Ra* = 100, d = 0.20.



(a) **(b)**

Fig. 7. Mean Nusselt **(a)** and Sherwood numbers **(b)** as a function of the source diameter. Da = 10^{-3}, Ra* = 100.

4.3. *Proposed models*

Summarizing our numerical predictions, (*see Fig. 8(a,b)*) and by taking into account the previous observations, new powerful correlations that give the heat and mass transfers into the porous square can be proposed as:

$$Sh\big|_{N \geq 0} = 0.7524 \left[\frac{Ra^{*}\ Le\ (N+1)}{(1-(d/H))^{3}} \right]^{0.3258} \qquad (R^2 = 0.998) \tag{10}$$

$$Nu\big|_{N>0} = 3.338 + 1.7144\ Log\left[\left(\frac{Ra^{*}|N|}{Le} (d/H)^{2.2} \right)^{0.48} \right] - 0.034 \left\{ Log\left[\left(\frac{Ra^{*}|N|}{Le} (d/H)^{2.2} \right)^{0.48} \right] \right\}^{2} \qquad (R^2 = 0.991) \tag{11}$$

The last ones, made using the solutal Rayleigh number Ra* Le (N+1) and the aspect ratio d/H, are available for a Lewis number ranging from 10 to 300, a buoyancy ratio taking between 0 and 50, a porous thermal Rayleigh number in between 100 and 2000, and an aspect ratio in between 0.20 and 0.80.

5. Conclusion

A numerical investigation of double-diffusive natural convection phenomenon inside a square Darcy-Brinkman porous enclosure, having a centered circular source was realized in this paper. In general; heat and mass transfers seems to be severely affected by the source diameter as well as the governing parameters such the buoyancy ratio, the Lewis and the porous thermal Rayleigh numbers. Then, new powerful Sherwood and Nusselt correlations, which display the heat and mass transfer inside the square, are expected with a precision of about 2%. Note that the present investigation doesn't take into account effects of other parameters such as the porosity and the Darcy number, what provide guidance for future work.

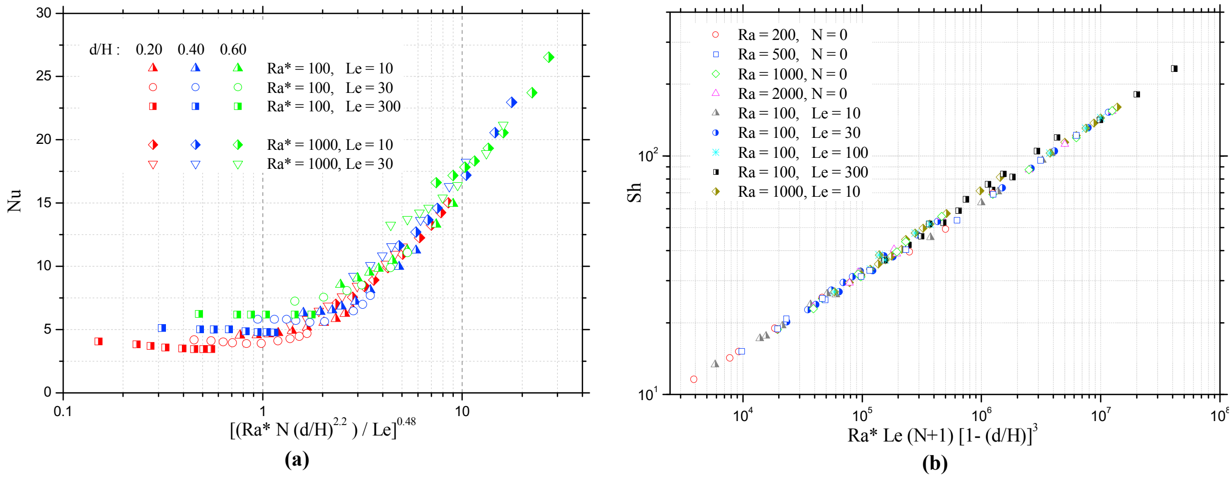


Fig.8. Proposed model for the mean Nusselt **(a)** and Sherwood numbers **(b)**.

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