Research Article Open Access

Abdelkader Boutra, Karim Ragui, Nabila Labsi, and Youb Khaled Benkahla*

Lid-Driven and Inclined Square Cavity Filled With a Nanofluid: Optimum Heat Transfer

DOI 10.1515/eng-2015-0028

Received September 16, 2014; accepted April 07, 2015

Abstract: This paper reports a numerical study on mixed convection within a square enclosure, filled with a mixture of water and Cu (or Ag) nanoparticles. It is assumed that the temperature difference driving the convection comes from the side moving walls, when both horizontal walls are kept insulated. In order to solve the general coupled equations, a code based on the finite volume method is used and it has been validated after comparison between the present results and those of the literature. To make clear the effect of the main parameters on fluid flow and heat transfer inside the enclosure, a wide range of the Richardson number, taken from 0.01 to 100, the nanoparticles volume fraction (0% to 10%), and the cavity inclination angle (0° to 180°) are investigated. The phenomenon is analyzed through streamlines and isotherm plots, with special attention to the Nusselt number.

Keywords: mixed convection, lid-driven cavity; Cu and Ag nanoparticles; water base fluid; finite volume method

Nomenclature

C_p	Specific heat, [J/kgK]
Gr	Grashof number, $Gr = g\beta\rho_f^2 \Delta T H^3/\mu_f^2$
H	Cavity height, [m]
k	Thermal conductivity, [W/mK]
N	Normal direction to the wall
Nu	Nusselt number
$p^\star$	Pressure [Pa]
P	Dimensionless pressure
Pr	Prandtl number, $Pr = C_p\mu_f/k_f$
Re	Reynolds number, $Re = \rho_f V_{lid} H/\mu_f$
Ri	Richardson number, $Ri = Gr/Re^2$
T	Temperature [K]
u, v	Velocity components [m/s]
U, V	Dimensionless velocity components
x, y	Cartesian coordinates [m]
X, Y	Dimensionless Cartesian coordinates

Greek letters

α	Thermal diffusivity [m^2/s]
β	Thermal expansion coefficient [1/K]
θ	Dimensionless temperature
μ	Viscosity [kg/ms]
ρ_f	Fluid density [kg/m^3]
ϕ	Nanoparticles volume fraction
ψ	Cavity inclination angle

Subscripts

c	Cold
h	Hot
nf	Nanofluid
s	Solid particles

1 Introduction

Heat convection of nanofluids, which are a mixture of nanoparticles in a base fluid such water and oil, has been recently an active field of research since they are used to improve heat transfer. Compared to other techniques for enhancing heat transfer in practical applications, nanofluids have the advantage of behaving like pure fluids because of the nanometric size of introducing solid parti-

Abdelkader Boutra, Karim Ragui, Nabila Labsi: Laboratory of Transfer Phenomenon, University of Sciences and Technology Houari Boumediene BP. 32 El Alia, 16111 Bab Ezzouar, Algiers, Algeria

***Corresponding Author: Youb Khaled Benkahla:** Laboratory of Transfer Phenomenon, University of Sciences and Technology Houari Boumediene BP. 32 El Alia, 16111 Bab Ezzouar, Algiers, Algeria, E-mail: youbenkahla@yahoo.fr

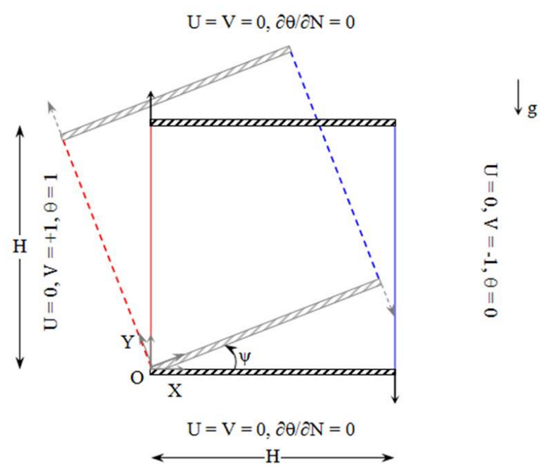

Figure 1: Simulation domain with its boundary conditions.

cles. Thus, they are used as heat transfer fluids for various applications, such as advanced nuclear systems or micro/mini channel heat sinks, electronic equipment as power transistors, printed wiring boards and chip packages mounted on computer mother boards.

To reveal the effects of such added nanoparticles on the heat transfer, some papers deal with natural convection of nanofluids, in differentially heated enclosures, were very helpful [1–3]. The literature indicates that most researches [4–6] were achieved using a closed configuration whilst a little attention has been devoted to the convection phenomenon within a moving wall enclosure [7, 8].

As such, the aim of the present paper is to examine the combined natural-forced convection heat transfer in a square cavity filled with a nanofluid in order to predict the effect of the Richardson number, the nanoparticles volume fraction and the square inclination angle on fluid flow and heat transfer as well.

Referred to papers of Elif [9], Santra *et al.* [10] and many others [11], the nanoparticles with a high thermal conductivity (such as Ag and Cu) produce a great enhancement in heat transfer rate. For this reason, Cu and Ag nanoparticules are considered in the present investigation.

It is to note that this kind of configurations represents an important industrial application such the shear mixture, where we train the fluid on both sides to generate a mixture.

Table 1: Thermo-physical properties of the base fluid and the nanoparticles at T = 25°C.

Thermo-physical properties	Fluid Phase (water)	Cu	Ag
C_p (J/kgK)	4179	385	230
ρ (kg/m^3)	997.1	8933	10500
k (W/mK)	0.613	401	418
β (1/K) 10^{-5}	21	1.67	1.97

2 Problem's statement and Mathematical formulation

The physical model of the problem, along with its boundary condition, is shown in Figure 1. A square inclined cavity filled with (Cu or Ag)-water nanofluid is considered, such as the water and the Cu (or Ag) nanoparticles are in thermal equilibrium. The side moving walls are maintained at a different constant temperatures θ_h and θ_c, respectively, when the horizontal walls are kept insulated. The flow and heat transfer are steady and twodimensional. The Richardson number (= Gr/Re^2) is taken as 0.01, 1 and 100 to simulate the forced, mixed and natural convection, respectively. The thermo-physical properties of the base fluid and the solid nanoparticles are given in Table 1. Constant thermo-physical properties are considered for the nanofluid, whereas the density variation in the buoyancy forces is determined using the Boussinesq approximation [12].

The density, ρ_{nf} , the heat capacity, $(\rho C_p)_{nf}$, the thermal expansion coefficient, $(\rho\beta)_{nf}$, and the thermal diffusivity of the nanofluid, α_{nf} , are defined, respectively, as follows [13]:

$$\rho_{nf} = (1-\phi)\rho_f + \phi\rho_s, \tag{1}$$

$$(\rho C_p)_{nf} = (1-\varphi)(\rho C_p)_f + \phi(\rho C_p)_s, \tag{2}$$

$$(\rho\beta)_{nf} = (1-\phi)(\rho\beta)_f + \phi(\rho\beta)_s, \tag{3}$$

$$\alpha_{nf} = \frac{k_{nf}}{(\rho C_p)_{nf}}. \tag{4}$$

The effective viscosity μ_{nf} and the thermal conductivity k_{nf} of the nanofluid are determined according to Brinkman [14], Equation 5, and Maxwell [15], Equation 6, models:

$$\mu_{nf} = \frac{\mu_f}{(1-\phi)^{2.5}}, \tag{5}$$

$$\frac{k_{nf}}{k_f} = \frac{(k_s + 2k_f) - 2\phi(k_s + k_f)}{(k_s + 2k_f) + \phi(k_s + k_f)}. \quad (6)$$

The dimensionless continuity, momentum (according to horizontal and vertical directions) and energy equations are given, respectively, as:

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0, \quad (7)$$

$$\begin{aligned} U\frac{\partial U}{\partial X} + V\frac{\partial U}{\partial Y} &= -\frac{\partial P}{\partial X} + \frac{1}{Re}\frac{\mu_{nf}}{\rho_{nf}\nu_f}\left[\frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2}\right] \\ &+ Ri\frac{(\rho\beta)_{nf}}{\rho_{nf}\beta_f}\theta\sin\psi, \end{aligned} \quad (8)$$

$$\begin{aligned} U\frac{\partial U}{\partial X} + V\frac{\partial V}{\partial Y} &= -\frac{\partial P}{\partial Y} + \frac{1}{Re}\frac{\mu_{nf}}{\rho_{nf}\nu_f}\left[\frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2}\right] \\ &+ Ri\frac{(\rho\beta)_{nf}}{\rho_{nf}\beta_f}\theta\cos\psi, \end{aligned} \quad (9)$$

$$U\frac{\partial \phi}{\partial X} + V\frac{\partial \phi}{\partial Y} = \frac{\alpha_{nf}}{\alpha_f}\frac{1}{PrRe}\left[\frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Y^2}\right], \quad (10)$$

where $Re = \rho_f V_{wall} H/\mu_f$ is the Reynolds number, $Ri = Gr/Re^2$ the Richardson number and $Pr = C_p\rho_f/k_f$ is the Prandtl number.

The mean Nusselt number over the hot (or cold) wall is given by the following expression:

$$|Nu|_{wall} = \int_0^1 \left(\frac{k_{nf}}{k_f}\right)\frac{\partial\theta}{\partial X}\bigg|_{X=0 \text{ or } 1} dY. \quad (11)$$

The mean Nusselt number inside the cavity is calculated as:

$$Nu = \frac{Nu_h + Nu_c}{2}. \quad (12)$$

3 Numerical procedure and validation

The governing equations are discretized in space using the finite volume method. As the momentum equation is formulated in terms of primitive variables (U, V and P), the iterative procedure includes a pressure correction calculation method, namely SIMPLER [16], to solve the pressure velocity coupling. In fact, a regular two-dimensional finite difference mesh is generated in the computational domain. Then, a square shaped control volume is generated around each nodal point. The governing equations are then integrated over each control volume. Subsequently, the derivatives of the dependent variables on the faces of the control volume in the resulting equations are replaced by finite difference forms written in terms of the nodal values of the dependent variables. A second-order central difference scheme is used for the diffusion terms while a hybrid scheme, a combination of upwind and central difference schemes, is employed for the convective terms citeb:16. Carrying out the same procedure for all the control volumes yields a system of algebraic equations with nodal values of the dependent variables as unknowns. The set of discretized equations are then solved iteratively yielding the velocity, pressure, and temperature at the nodal points. An under relaxation scheme is employed to obtain converged solutions.

The adopted convergence criterion for the temperature, the pressure, and the velocity is given as:

$$\frac{\sum_{j=2}^{m}\sum_{i=1}^{n}\left|\phi_{i.j}^{\zeta+1} - \phi_{i,j}^{\zeta}\right|}{\sum_{j=1}^{m}\sum_{i=1}^{n}\left|\phi_{i,j}^{\zeta+1}\right|} \leq 10^6, \quad (13)$$

where both m and n are the numbers of grid points in the x-and y-directions,respectively, ϕ is any of the computed field variables, and ζ is the iteration number.

The performance of the using code via the nanofluid convection problem in a confined enclosure is established by comparing predictions with other numerical results and by verifying the grid independence of the present results. First, the present results are consistent with previous computations, namely those of Oztop and Abu-Nada [17] as shown in Table 2. The present results and those reported by Oztop and Abu-Nada are in excellent agreement since the maximum difference between both results is about 2%. Then, in order to determine an appropriate grid for the numerical simulations, a grid independence study is conducted for the different values of the Richardson number (0.01, 1 and 100) as shown in Table 3. The calculations are performed for the Cu-water nanofluid of ϕ equals 0.02 with six different uniform mesh grids: 51^2, 71^2, 91^2, 101^2, 111^2, and 121^2. It is observed that a 91^2 uniform mesh grid is adequate for a grid independent solution. However, a fine structured mesh of 101^2 is used to avoid round-off error for all other calculations in this investigation.

4 Results and discussion

First of all, we investigate the convection phenomenon inside the square Cu (or Ag)-water nanofluid cavity, of an inclination angle equals 0°. The Richardson number is ad-

Table 2: Average Nusselt number of the heat source compared with Oztop and Abu-Nada [17].

Ra	ϕ	Oztop and Abu-nada [17]	Present study	Relative gap (%)
	0.00	1.004	1.005	0.10
	0.05	1.122	1.128	0.53
10^3	0.10	1.251	1.248	0.24
	0.15	1.423	1.419	0.28
	0.20	1.627	1.621	0.37
	0.00	2.010	2.032	1.08
	0.05	2.122	2.109	0.61
10^4	0.10	2.203	2.199	0.18
	0.15	2.283	2.291	0.35
	0.20	2.363	2.382	0.80
	0.00	3.983	3.992	0.22
	0.05	4.271	4.256	0.35
10^5	0.10	4.440	4.389	1.16
	0.15	4.662	4.680	0.38
	0.20	4.875	4.861	0.29

Table 3: Average Nusselt number of the Cu-water nanofluid for different uniform grids. $\phi = 0.02$, $Pr = 6.2$.

nodes	Nu		
Number	$Ri = 0.1$	$Ri = 1$	$Ri = 100$
51×51	36.3109	16.0545	5.6052
71×71	43.7220	17.4163	5.8391
91×91	49.9145	17.9306	5.9436
101×101	**53.1219**	**19.9059**	**6.2693**
111×111	53.1216	19.9063	6.2697
121×121	53.1220	19.9062	6.2695

justed between 0.01 and 100 when the nanofluid volume fraction is fixed at 0.02. The streamlines and the isotherm patterns for various values of the Richardson number are presented in Figure 2.

In the case of a dominant forced convection ($Ri = 0.01$), we note a single clockwise rotation cell transported by the translational movement of the active walls. The bottom left and the top right regions seem to be less stirred by the flow; this behavior disappears gradually by increasing the Richardson number.

With mixed convection ($Ri = 1$), we denote a combined effect of the translational motion of the active walls as well as that introduced by the thermal buoyancy forces what causes, then, an extension lengthening of the cell along the diagonal. The latter becomes horizontally extended when a dominated natural convection ($Ri = 100$) is considered, since the active walls effect is weak compared to the thermal buoyancy forces. In the vicinity of the horizontal and the vertical walls, the flow is generally unidirectional. The Isotherm plots are also presented as a function of the Richardson number, as displayed in Figure 2(b). In the vicinity of the active walls, the thermal boundary layers are found to be less important with the increase of the Richardson number. With the mixed convection case, great temperature gradients are localized in the lower-left and the upper-right parts of the active walls, with a penetration of the hot fluid within the cavity from the upper side, (and vice versa for the cold fluid). At $Ri = 100$, the penetration of the heat into the center of the cavity is still observed, whilst the isotherms become fully horizontal.

The effect of the nanoparticles volume fraction on the fluid flow and heat transfer has also been investigated as plotted in Figure 3. Unlike the streamlines, where the same fluid motion is observed, it can be seen from the isotherms that the increase of the latter to 10% leads to the decrease of the thermal boundary layers thickness to be more compact near the active walls. This behavior is reflected in the mean Nusselt number variations according to the Cu and Ag nanoparticles volume fraction, for different values of the Richardson number, shown in Figure 4. For both nanofluids, the last is found to be as a linear increasing function of the nanoparticles volume fraction and decreases with the increase Richardson number. It is worth noting that far from the natural convection mode, natural convection ($Ri = 100$), the heat transfer is more important for the Ag-water nanofluid compared to the Cu-water one, what makes the use of the Ag nanoparticles more required.

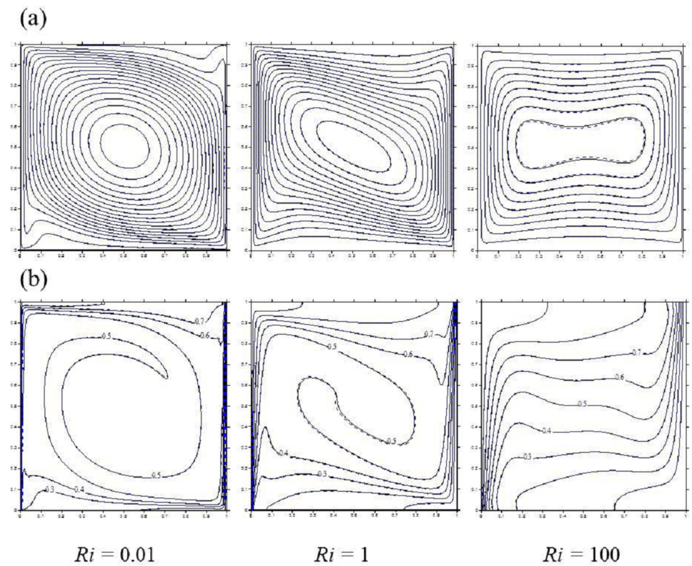

Figure 2: Streamlines (a) and Isotherms (b) for different values of the Richardson number. $\phi = 0.02$, [—- Cu-water, — Ag-water].

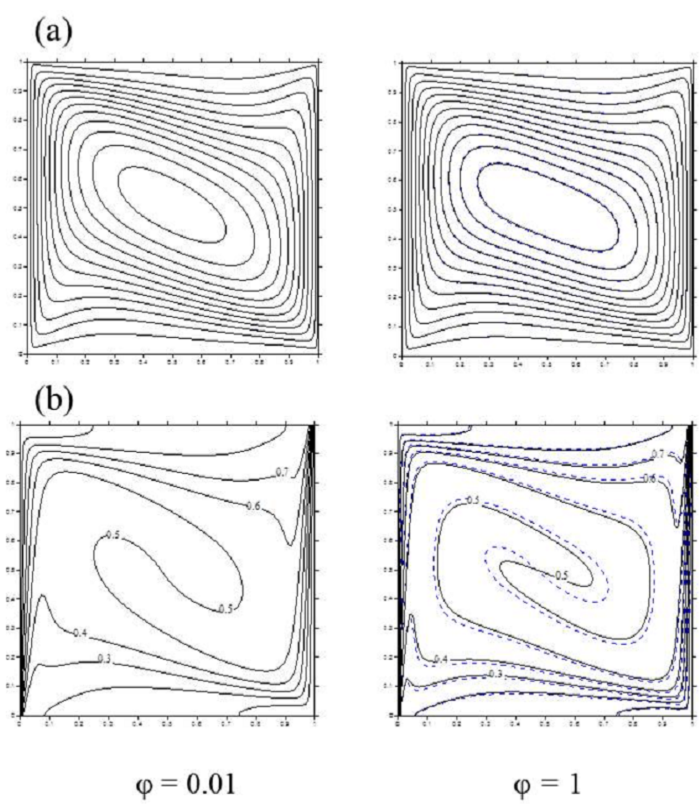

Figure 3: Streamlines (a) and Isotherms (b) of the nanofluids for different values of the solid volume fraction. $Ri = 1$,[— Cu-water, — Ag-water].

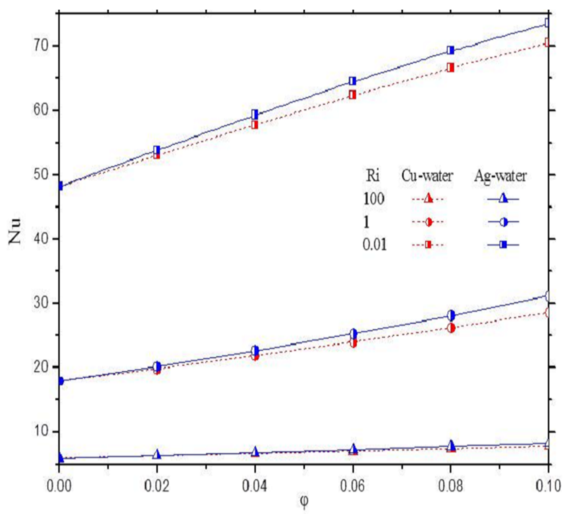

Figure 4: Variation of the mean Nusselt number with the nanoparticles volume fraction for different values of Ri.

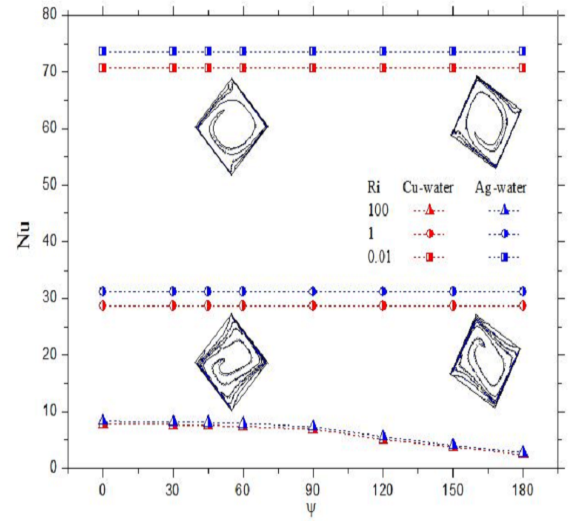

Figure 5: Nusselt number as a function of ψ for different values of Ri.

Regarding, the cavity inclination angle effect, Figure 5 displays the mean Nusselt number as a function of the latter (noted as ψ) and that, for different values of the Richardson number. As we can see and except for the natural convection mode, the increase of the cavity inclination angle has no significant effect on heat transfer, since the maximum enhancement is found to be quietly over 0.2% than that calculated into a non-inclined one.

However, for $Ri = 100$, the increase of the cavity inclination angle over 90° leads to the decrease of the heat transfer rate to reach 69% for an inclination angle of 180°, compared to the case of a non-inclined one. To explain this phenomenon, Figure 6 displays both the streamlines and the isotherm plots for different inclination angles, ranging from 90° to 180°. It can be seen the appearance of a mono-cellular structure when the inclination angle ranges between 90° and 95°. At 96° we denote the destruction of this structure and the appearance of a central cell, with two others near the active walls. By increasing the cavity inclination angle, the central cell becomes more important to the detriment of the wall cells. This can be explained by the fact that the vertical component of the wall velocity increases, and acting strongly in the opposite direction of the buoyancy forces. Consequently, the hot fluid remains confined near the hot wall when the cold fluid near the cold wall. As a result, the extent of the wall cells decreases and so do the mean Nusselt number (Figure 5) when the wall temperature gradient decreases as shown in the Isotherm plots (Figure 6). The minimum heat transfer rate is obtained for 180°, where the active walls' motion is in the opposite direction with the buoyancy forces (upward cold and downward cold walls).

5 Conclusion

The analysis of the mixed convection phenomenon in a side moving walls cavity, filled with a nanofluid, was realized through our paper. Taking into account the effects of various parameters, such the Richardson number, the nanoparticles volume fractions, and the inclination angle, the results may resume as follows:

1. Heat transfer is a decreasing function of Richardson number.
2. Heat transfer is improved by the addition of nanoparticules to the base fluid. This enhancement is more pronounced by the increase of the nanoparticles volume fraction.
3. The type of the nanoparticles is a key factor for heat transfer enhancement. Indeed, the highest values of the mean Nusselt number are obtained for Ag nanoparticles.
4. Regarding the forced and mixed convection modes, the inclination angle has no significant effect on the heat transfer.
5. In the case of the natural convection mode, and over 90°, the inclination angle of the enclosure decreases the heat transfer, the reduction is about 69% compared to the case of a non-inclined one.

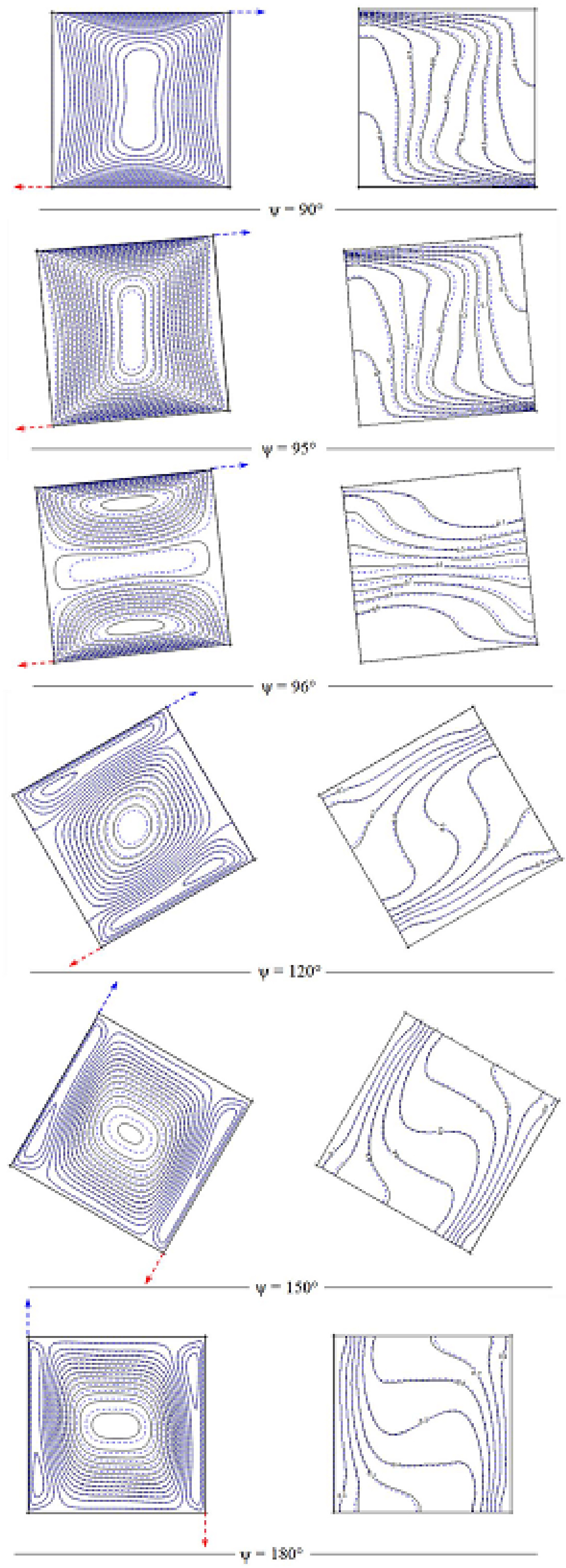

Figure 6: Streamlines (a) and Isotherm plots (b) of the nanofluids for different values of the inclination angle. $\phi = 0.10$, $Ri = 100$, [—- Cu-water, — Ag-water].

References

[1] Choi S. U. S., Enhancing thermal conductivity of fluids with nanoparticles. ASME Fluids Engineering Division, 1995, 231, 99-105.

[2] Wang X., Xu X., Choi S. U. S., Thermal conductivity of nanoparticles fluid mixture. Journal of Thermophysics and Heat Transfer, 1999, 13(4), 474-480.

[3] Jou R. Y., Tzeng S. C., Numerical research of nature convective heat transfer enhancement filled with nanofluids in rectangular enclosures. International Communications in Heat and Mass Transfer, 2006, 33, 727-736.

[4] He Y., Men Y., Zhao Y., Lu H., Ding Y., Numerical investigation into the convective heat transfer of TiO2 nanofluids flowing through a straight tube under the laminar flow conditions. Applied Thermal Engineering, 2009, 29, 1965-1972.

[5] Ashorynejad H. R., Mohamad A. A., Sheikholeslami M., Magnetic field effects on natural convection flow of a nanofluid in a horizontal cylindrical annulus using Lattice Boltzmann method. International Journal of Thermal Science, 2013, 64, 240-250.

[6] Akbarinia A., Behzadmehr A., Numerical study of laminar mixed convection of a nanofluid in horizontal curved tubes. Applied Thermal Engineering, 2007, 27, 1327-1337.

[7] Arani A., Mazrouei S., Mahmoodi M., Numerical study of mixed convection flow in a lid-driven cavity with sinusoidal heating on sidewalls using nanofluid. Super-lattices and Microstructures, 2012, 51, 893-911.

[8] Ghasemi B., Aminossadati S. M., Mixed convection in a lid-driven triangular enclosure filled with nanofluids. International Communications in Heat and Mass Transfer, 2010, 37, 1142-1148.

[9] Santra A. K., Sen S., Chakraborty N., Study of heat transfer augmentation in a differentially heated square cavity using copper-water nanofluid. International Journal of Thermal Science, 2008, 47, 1113-1122.

[10] Elif B. O., Natural convection of water-based nanofluids in an inclined enclosure with a heat source. International Journal of Thermal Science, 2009, 48, 2063-2073.

[11] Ghasemi B., Aminossadati S. M., Natural convection heat transfer in an inclined enclosure filled with a CuO-water nanofluid. Numerical Heat Transfer Part A, 2009, 07, 807-823.

[12] Bejan A., Convection heat transfer, John Wiley & Sons, Inc., Hoboken, New jersey, USA, 2004.

[13] Khanafer K., Vafai K., Lightstone M., Buoyancy-driven heat transfer enhancement in a two-dimensional enclosure utilizing nanofluids. International Journal of Heat and Mass Transfer, 2003, 6, 3639-3653.

[14] Brinkman H. C., The viscosity of concentrated suspensions and solutions. Journal of Chemical Physics, 1952, 20, 571-581

[15] Maxwell J. C., A Treatise on Electricity and Magnetism Vol. II, Oxford University Press Cambridge U.K, 1873.

[16] Patankar V., Numerical Heat transfer and fluid flow, Hemisphere Publishing Corporating, Taylor and Francis Group, New York, 1980.

[17] Oztop H. F., Abu-Nada, E., Numerical study of natural convection in partially heated rectangular enclosures filled with nanofluids. International Journal of Heat and Fluid Flow, 2008, 29, 1326-1336.